

# AN EXPERIMENTAL CASE STUDY ON UNCERTAINTY QUANTIFICATION IN STRUCTURAL DYNAMICS

S. Adhikari, K. Lonkar and M. I. Friswell

Department of Aerospace Engineering, University of Bristol, Queens Building, University Walk, Bristol BS8 1TR (U.K.)

## ABSTRACT

The consideration of uncertainties in numerical models to obtain the probabilistic descriptions of vibration response is becoming more desirable for industrial scale finite element models. Broadly speaking, there are two 'parts' in this problem. The first is the quantification of parametric and nonparametric uncertainties associated with the model and the second is the propagation of uncertainties through it. While the second part has been extensively researched in the past two decades (e.g., the stochastic finite element method), it is only relatively recently the first part is being considered seriously. This paper considers the 'first part' and is aimed at gaining more insights into the nature of uncertainties in medium and high frequency vibration problems. Results from an experimental study that may be used for this purpose are discussed in detail. The experiment is on a fixed-fixed beam with twelve masses placed at random locations. The total amount of 'random masses' is less than 2% of the total mass of the beam. This experiment is aimed at simulating 'random errors' in the mass matrix. The probabilistic characteristics of the frequency response functions are discussed in the low, medium and high frequency ranges.

## 1 INTRODUCTION

The steady development of powerful computational hardware in recent years have led to high-resolution finite element models of real-life engineering structural systems. However, for high-fidelity and credible numerical models, the high-resolution in the numerical grid is not enough. It is also required to quantify uncertainties and robustness associated with a numerical model. As a result, the quantification of uncertainties plays a key role in establishing the credibility of a numerical model. Uncertainties can be broadly divided into two categories. The first type is due to the inherent variability in the system parameters, for example, different cars manufactured from a single production line are not exactly the same. This type of uncertainty is often referred to as *aleatoric uncertainty*. If enough samples are present, it is possible to characterize the variability using well established statistical methods and consequently the probability density functions (pdf) of the parameters can be obtained. The second type of uncertainty is due to the lack of knowledge regarding a system, often referred to as *epistemic uncertainty*. This kind of uncertainty generally arises in the modelling of complex systems, for example, in the modeling of cabin noise in helicopters. Due to its very nature, it is comparatively difficult to quantify or model this type of uncertainty. There are two broad approaches to quantify uncertainties in a model. The first is the *parametric approach* and the second is the *non-parametric approach*. In the parametric approach the uncertainties associated with the system parameters, such as Young's modulus, mass density, Poisson's ratio, damping coefficient and geometric parameters are quantified using statistical methods and propagated, for example, using the stochastic finite element method [1–9]. This type of approach is suitable to quantify aleatoric uncertainties. Epistemic uncertainty on the other hand does not explicitly depend on the system parameters. For example, there can be unquantified errors associated with the equation of motion (linear or non-linear), in the damping model (viscous or non-viscous), in the model of structural joints, and also in the numerical methods (e.g. discretisation of displacement fields, truncation and roundoff errors, tolerances in the optimization and iterative algorithms, step-sizes in the time-integration methods). It is evident that the parametric approach is not suitable to quantify this type of uncertainty. As a result non-parametric approaches [10–15] have been proposed for this purpose. This type of approach has recently been applied in conjunction with experimental measurements [16, 17], non-linear systems [18, 19] and to model data and model uncertainties in complex aerospace structural systems [20].

Uncertainties associated with a variable can be characterized using the probabilistic approach or possibilistic approaches based

on interval algebra, convex sets or Fuzzy sets. In this paper the probabilistic approach has been adopted. The equation of motion of a damped  $n$ -degree-of-freedom linear structural dynamic system can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \quad (1)$$

where  $\mathbf{M} \in \mathbb{R}_{n,n}$ ,  $\mathbf{C} \in \mathbb{R}_{n,n}$  and  $\mathbf{K} \in \mathbb{R}_{n,n}$  are the mass, damping and stiffness matrices respectively. The importance of considering parametric and/or non-parametric uncertainty also depends on the frequency of excitation. For example, in high frequency vibration the wave lengths of the vibration modes become very small. As a result the vibration response can be very sensitive to the small details of the system. In such situations a non-parametric uncertainty model may be adequate. Overall, three *different* approaches are currently available to analyze stochastic structural dynamic systems across the frequency range:

- *Low frequency vibration problems*: Stochastic Finite Element Method [1–9, 21] (SFEM) - considers parametric uncertainties in detail;
- *High frequency vibration problems*: Statistical Energy Analysis [22] (SEA) - does not consider parametric uncertainties in detail;
- *Mid-frequency vibration problems* [23–29]: both parametric and non-parametric uncertainties need to be considered.

This paper describes a experiment that may be used to test methods of uncertainty quantification across the frequency range. The difference between this data and previous experimental data is that the tests are closely controlled and the uncertainty is 'known'. This allows to model uncertainty, propagate it through dynamical models and compare the results with this experimentally obtained data. The experiment is on a fixed-fixed beam with randomly placed masses. One hundred experiments have been carried out to obtain the statistics of the measured response in low, medium and high frequency ranges.

## 2 UNCERTAINTY QUANTIFICATION IN A FIXED-FIXED BEAM

### 2.1 System Model and Experimental Setup

A steel beam with uniform rectangular cross-section is used for the experiment. The physical and geometrical properties of the steel beam are shown in Table 1. The beam used here is actually a 1.5m ruler made of mild steel. A ruler was used for the ease

Beam Properties	Numerical values
Length ( $L$ )	1200 mm
Width ( $b$ )	40.06 mm
Thickness ( $t_h$ )	2.05 mm
Mass density ( $\rho$ )	7800 Kg/m <sup>3</sup>
Young's modulus ( $E$ )	$2.0 \times 10^5$ MPa
Cross sectional area ( $a = bt_h$ )	$8.212 \times 10^{-5}$ m <sup>2</sup>
Moment of inertia ( $I = 1/12bt_h^3$ )	$2.876 \times 10^{-11}$ m <sup>4</sup>
Mass per unit length ( $\rho_l$ )	0.641 Kg/m
Bending rigidity ( $EI$ )	5.752 Nm <sup>2</sup>
Total weight	0.7687 Kg

**TABLE 1: Material and geometric properties of the beam considered for the experiment**

of placing the masses at predetermined locations. The ruler was clamped between 0.05m and 1.25m so that the effective length of the vibrating beam is 1.2m. The overall experiential setup is shown in Figure 1. The end clamps were screwed into two heavy steel blocks which in turn were fixed to a table.

Twelve equal masses are used to simulate a randomly varying mass distribution. The masses are actually magnets so that they can be attached easily anywhere in the steel beam. These magnets are cylindrical in shape and 12.0 mm in length and 6.0 mm in diameter. Some of the attached masses for a sample realization are shown in Figure 2. Each of them weights 2g so that the total amount of variable mass is 1.6% of the mass of the beam. The location of the 12 masses are assumed to be between 0.2m and 1.0m of the beam. A uniform distribution with 100 samples is used to generate the mass location. The mean and the



Figure 1: The test rig for the fixed-fixed beam.

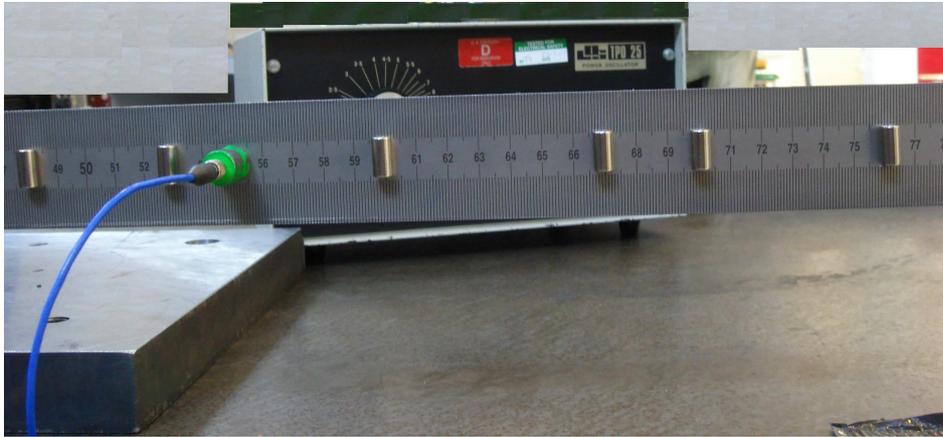


Figure 2: Attached masses (magnets) at random locations. In total 12 masses, each weighting 2g, are used.

standard deviation of the mass-location are given by

$$\bar{x}_m = [0.2709, 0.3390, 0.3972, 0.4590, 0.5215, 0.5769, 0.6398, 0.6979, 0.7544, 0.8140, 0.8757, 0.9387] \quad (2)$$

$$\text{and } \sigma_{x_m} = [0.0571, 0.0906, 0.1043, 0.1034, 0.1073, 0.1030, 0.1029, 0.1021, 0.0917, 0.0837, 0.0699, 0.0530] \quad (3)$$

Examples of the variation of the locations of the twelve masses are shown in Figure 3.

## 2.2 Experimental Methodology

A 32 channel LMS™ system is used to run the experiment. Three main components of the implemented experiment technique are (a) excitation of the structure, (b) sensing of the response, and (c) data acquisition and processing. In this experiment we used a shaker (the make, model no. and serial no are respectively, LDS V201, and 92358.3) to act as an impulse hammer. The problem with using the usual manual hammer is that it is in general difficult to hit the beam exactly at the same point with the same amount of force for every sample run. The shaker generates impulses at a pulse rate of 20s and a pulse width of 0.01s. Using the shaker in this way we have tried to eliminate any uncertainties arising from the input forces. This innovative experimental technique is designed to ensure that the resulting uncertainty in the response arises purely due to the random locations of the attached masses. Figure 4 shows the arrangement of the shaker. We have used a small circular brass plate weighting 2g to take impact from the shaker. This is done in order to obtain the driving point frequency response function. The details of the force transducer attached to the shaker is given in Table 2.

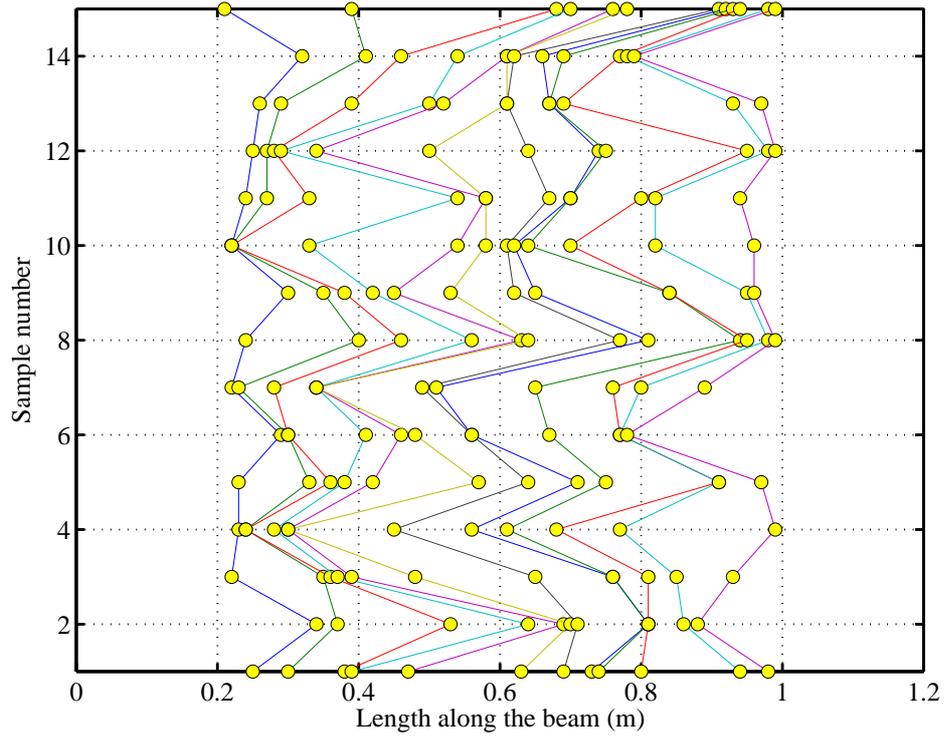


Figure 3: First 15 samples of the locations of 12 masses along the length of the beam.

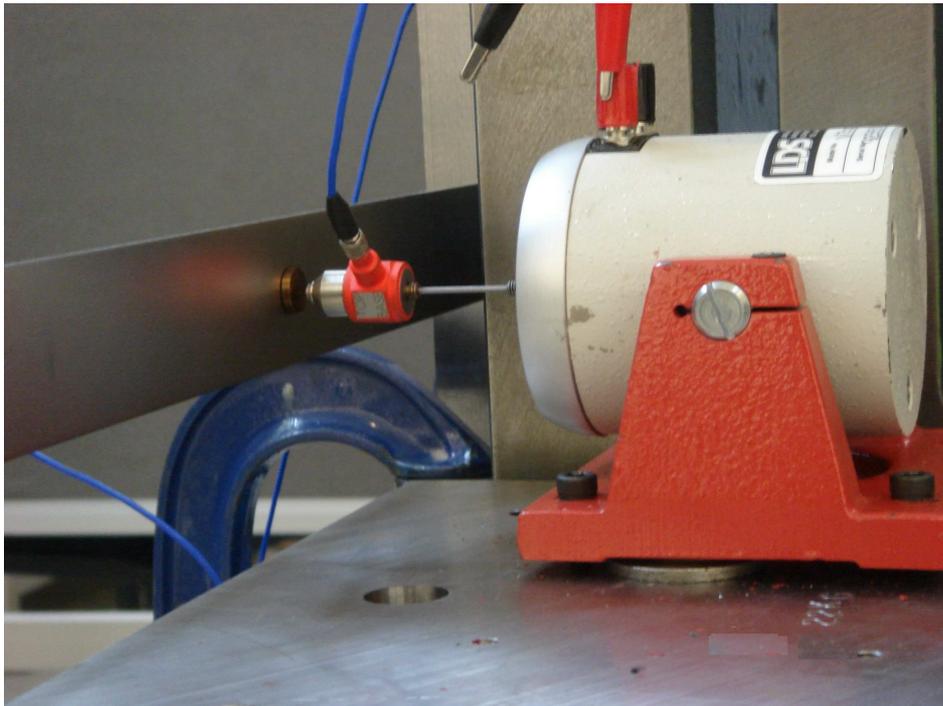


Figure 4: The shaker used as an impulse hammer using Simulink™. A hard steel tip used

In this experiment three accelerometers are used as the response sensors. The locations of the three sensors are selected such that two of them are near the two ends of the beam and one is near the middle of the beam. The exact locations are calculated such that the nodal lines of the first few bending modes can be avoided. The details of the accelerometers, including

their locations, are shown in Table 2. Small holes were drilled into the beam and all of the three accelerometers were attached

Role	Model & Serial number	Position from the left end	LMS channel	Sensitivity
Sensor (accelerometer)	PCB 333M07 SN 25948	23 cm (Point1)	1	98.8 mV/g
Sensor (accelerometer)	PCB 333M07 SN 26018	50 cm (Point2)	2	101.2 mV/g
Sensor (accelerometer)	PCB 333M07 SN 25942	102 cm (Point3)	3	97.6 mV/g
Actuator (force transducer)	PCB 208C03 21487	50 cm (Point2)	4	2.24 mV/N

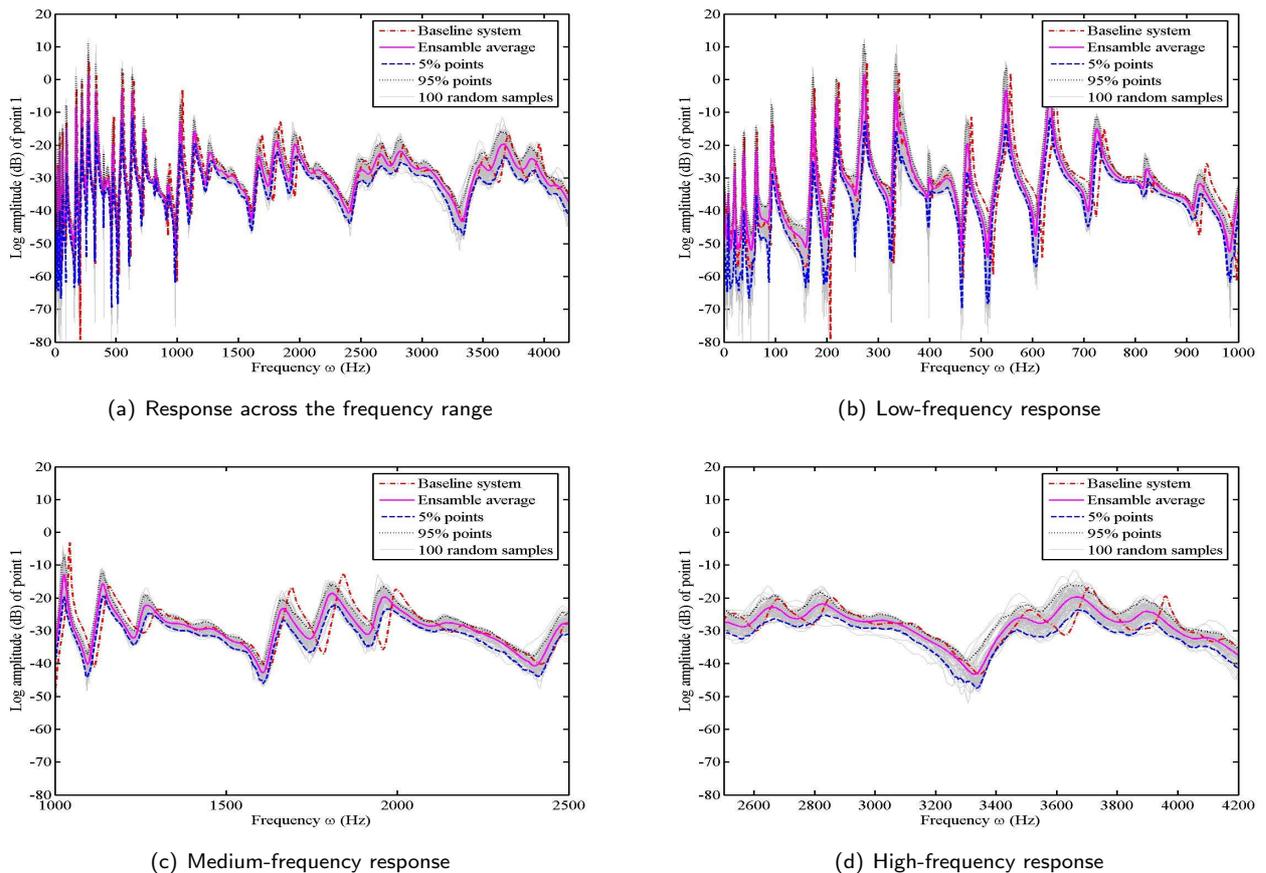
**TABLE 2: The details of the accelerometers and the shaker.**

by screwing through the holes.

The signal from the force transducer was amplified using a KISTLER type 5134 amplifier (with settings Gain: 100, Filter: 10K and Bias: Off) while the signals from the accelerometers were directly input into the LMS system. For data acquisition and processing, LMS Test Lab 5.0 was used. In Impact Scope, we have to set the bandwidth to 8192 Hz with 8192 spectral lines (i.e., 1.00 Hz resolution). The steel tip used in the experiment only gives clean data up to approximately 4500 Hz. As a result we have used 4.2 KHz as the upper limit of the frequency in the measured frequency response functions.

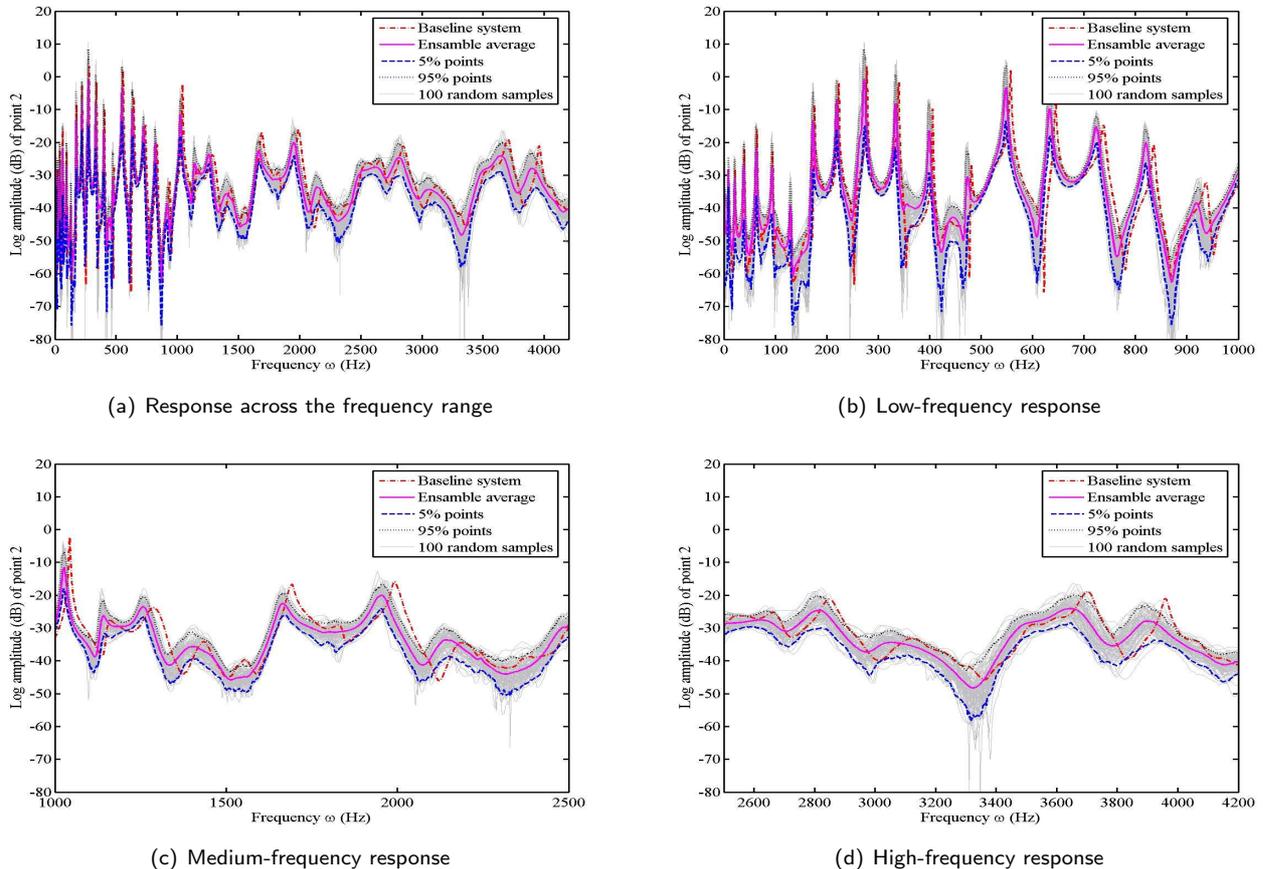
### 2.3 Results and Discussions

Figure 5 shows the amplitude of the frequency response function (FRF) at point 1 (see Table 2) of the beam without any masses (the baseline model). In the same figure 100 samples of of the amplitude of the FRF are shown together with the ensemble mean, 5% and 95% probability lines. In figures 5(b)-(d) we have separately shown the low, medium and high-frequency



**Figure 5: Amplitude of the FRF of the beam at point 1 (23 cm from the left end) with 12 randomly placed masses. 100 FRFs, together with the ensemble mean, 5% and 95% probability points are shown.**

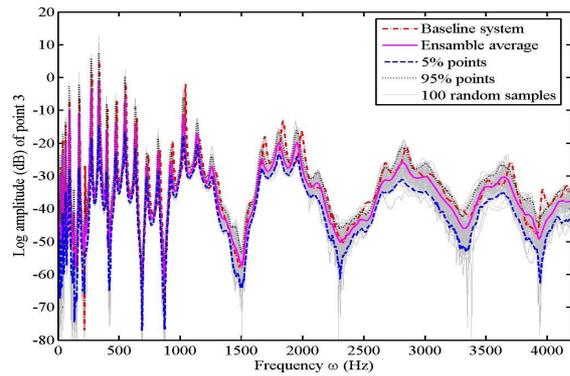
response, obtained by zooming around the appropriate frequency ranges in Figure 5(a). There are of course no fixed and definite boundaries between the low, medium and high-frequency ranges. Here we have selected 0 – 1.0KHz as the low-frequency vibration, 1.0 – 2.5KHz as the medium-frequency vibration and 2.5 – 4.2KHz as the high-frequency vibration. These frequency boundaries are selected on the basis of the qualitative nature of the response and devised purely for the purpose of the presentation of our results. The experimental approach discussed here is independent on these selections. The ensemble mean follows the result of the baseline system closely only in the low frequency range. The relative variance of the amplitude of the FRF remains more or less constant in the mid and high frequency range. Equivalent results for point 2 (the driving point FRF, see Table 2) and point 3 are shown in Figure 6 and Figure 7 respectively.



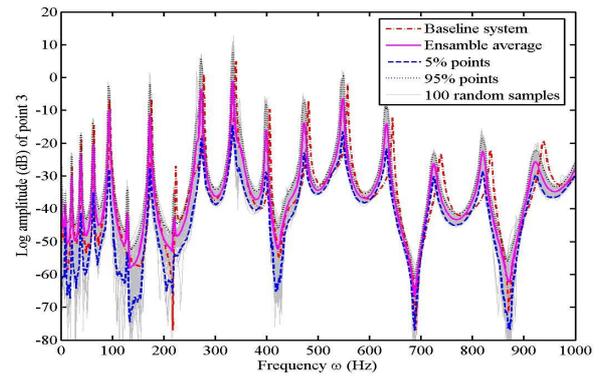
**Figure 6: Amplitude of the FRF of the beam at point 2 (the driving point FRF, 50 cm from the left end) with 12 randomly placed masses. 100 FRFs, together with the ensemble mean, 5% and 95% probability points are shown.**

### 3 CONCLUSIONS

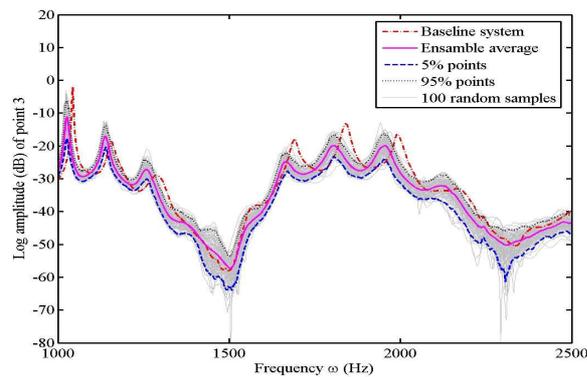
This paper has described a experiment that may be used to study methods to quantify uncertainty in the dynamics of structures. The fixed-fixed beam is very easy to model and the results of a 100 sample experiment with randomly placed masses were described in this paper. Special care has been taken so that the uncertainty in the response only arises from the randomness in the mass locations. Statistics of the frequency response function measured at three points of the beam were obtained for low, medium and high frequency ranges. It is expected that this data can be used for model validation and uncertainty quantification of dynamical systems.



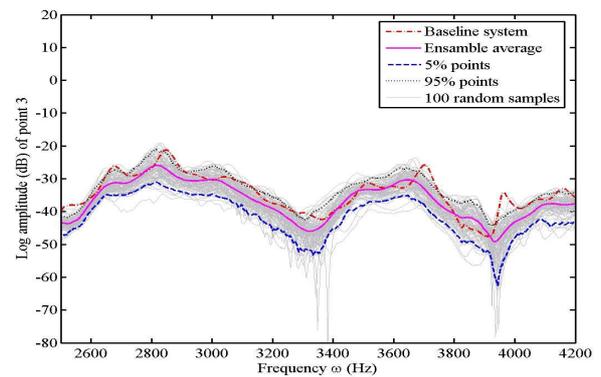
(a) Response across the frequency range



(b) Low-frequency response



(c) Medium-frequency response



(d) High-frequency response

Figure 7: Amplitude of the FRF of the beam at point 3 with 12 randomly placed masses (102 cm from the left end). 100 FRFs, together with the ensemble mean, 5% and 95% probability points are shown.

## REFERENCES

- [1] **Shinozuka, M.** and **Yamazaki, F.**, *Stochastic finite element analysis: an introduction, Stochastic structural dynamics: Progress in theory and applications*, edited by Ariaratnam, S. T., Schueller, G. I. and Elishakoff, I., Elsevier Applied Science, London, 1998.
- [2] **Ghanem, R.** and **Spanos, P.**, *Stochastic Finite Elements: A Spectral Approach*, Springer-Verlag, New York, USA, 1991.
- [3] **Kleiber, M.** and **Hien, T. D.**, *The Stochastic Finite Element Method*, John Wiley, Chichester, 1992.
- [4] **Manohar, C. S.** and **Adhikari, S.**, *Dynamic stiffness of randomly parametered beams*, Probabilistic Engineering Mechanics, Vol. 13, No. 1, pp. 39–51, January 1998.
- [5] **Adhikari, S.** and **Manohar, C. S.**, *Dynamic analysis of framed structures with statistical uncertainties*, International Journal for Numerical Methods in Engineering, Vol. 44, No. 8, pp. 1157–1178, 1999.
- [6] **Adhikari, S.** and **Manohar, C. S.**, *Transient dynamics of stochastically parametered beams*, ASCE Journal of Engineering Mechanics, Vol. 126, No. 11, pp. 1131–1140, November 2000.
- [7] **Haldar, A.** and **Mahadevan, S.**, *Reliability Assessment Using Stochastic Finite Element Analysis*, John Wiley and Sons, New York, USA, 2000.
- [8] **Sudret, B.** and **Der-Kiureghian, A.**, *Stochastic Finite Element Methods and Reliability*, Tech. Rep. UCB/SEMM-2000/08, Department of Civil & Environmental Engineering, University Of California, Berkeley, November 2000.
- [9] **Elishakoff, I.** and **Ren, Y. J.**, *Large Variation Finite Element Method for Stochastic Problems*, Oxford University Press, Oxford, U.K., 2003.
- [10] **Soize, C.**, *A nonparametric model of random uncertainties for reduced matrix models in structural dynamics*, Probabilistic Engineering Mechanics, Vol. 15, No. 3, pp. 277–294, 2000.

- [11] **Soize, C.**, *Maximum entropy approach for modeling random uncertainties in transient elastodynamics*, Journal of the Acoustical Society of America, Vol. 109, No. 5, pp. 1979–1996, May 2001, part 1.
- [12] **Soize, C.**, *Transient responses of dynamical systems with random uncertainties*, Probabilistic Engineering Mechanics, Vol. 16, No. 4, pp. 363–372, 2001.
- [13] **Soize, C.**, *Random matrix theory for modeling uncertainties in computational mechanics*, Computer Methods in Applied Mechanics and Engineering, Vol. 194, No. 12-16, pp. 1333–1366, 2005.
- [14] **Soize, C.**, *Non-Gaussian positive-definite matrix-valued random fields for elliptic stochastic partial differential operators*, Computer Methods in Applied Mechanics and Engineering, Vol. 195, No. 1-3, pp. 26–64, 2006.
- [15] **Adhikari, S.**, *A non-parametric approach for uncertainty quantification in elastodynamics*, 47th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics & Materials Conference, Newport, Rhode Island, USA, May 2006.
- [16] **Arnst, M., Clouteau, D., Chebli, H., Othman, R. and Degrande, G.**, *A non-parametric probabilistic model for ground-borne vibrations in buildings*, Probabilistic Engineering Mechanics, Vol. 21, No. 1, pp. 18–34, 2006.
- [17] **Chebli, H. and Soize, C.**, *Experimental validation of a nonparametric probabilistic model of nonhomogeneous uncertainties for dynamical systems*, Journal of the Acoustical Society of America, Vol. 115, No. 2, pp. 697–705, 2004.
- [18] **Desceliers, C., Soize, C. and Cambier, S.**, *Non-parametric-parametric model for random uncertainties in non-linear structural dynamics: Application to earthquake engineering*, Earthquake Engineering & Structural Dynamics, Vol. 33, No. 3, pp. 315–327, 2004.
- [19] **Desceliers, C. and Soize, C.**, *Non-linear viscoelastodynamic equations of three-dimensional rotating structures in finite displacement and finite element discretization*, International Journal of Non-Linear Mechanics, Vol. 39, No. 3, pp. 343–368, 2004.
- [20] **Capiez-Lernout, E., Pellissetti, M., Pradlwarter, H., Schueller, G. I. and Soize, C.**, *Data and model uncertainties in complex aerospace engineering systems*, Journal of Sound and Vibration, 2006, published online.
- [21] **Matthies, H. G., Brenner, C. E., Bucher, C. G. and Soares, C. G.**, *Uncertainties in probabilistic numerical analysis of structures and solids - Stochastic finite elements*, Structural Safety, Vol. 19, No. 3, pp. 283–336, 1997.
- [22] **Lyon, R. H. and Dejong, R. G.**, *Theory and Application of Statistical Energy Analysis*, Butterworth-Heinmann, Boston, 2nd edn., 1995.
- [23] **Langley, R. S. and Bremner, P.**, *A hybrid method for the vibration analysis of complex structural-acoustic systems*, Journal of the Acoustical Society of America, Vol. 105, No. 3, pp. 1657–1671, March 1999.
- [24] **Sarkar, A. and Ghanem, R.**, *Mid-frequency structural dynamics with parameter uncertainty*, Computer Methods in Applied Mechanics and Engineering, Vol. 191, No. 47-48, pp. 5499–5513, 2002.
- [25] **Soize, C. and Mziou, S.**, *Dynamic substructuring in the medium-frequency range*, AIAA Journal, Vol. 41, No. 6, pp. 1113–1118, 2003.
- [26] **Sarkar, A. and Ghanem, R.**, *A substructure approach for the midfrequency vibration of stochastic systems*, Journal of the Acoustical Society of America, Vol. 113, No. 4, pp. 1922–1934, 2003, part 1.
- [27] **Sarkar, A. and Ghanem, R.**, *Reduced models for the medium-frequency dynamics of stochastic systems*, Journal of the Acoustical Society of America, Vol. 113, No. 2, pp. 834–846, 2003.
- [28] **Soize, C. and Chebli, H.**, *Random uncertainties model in dynamic substructuring using a nonparametric probabilistic model*, Journal of Engineering Mechanics, ASCE, Vol. 129, No. 4, pp. 449–457, 2003.
- [29] **Soize, C.**, *Uncertain dynamical systems in the medium-frequency range*, Journal of Engineering Mechanics, ASCE, Vol. 129, No. 9, pp. 1017–1027, 2003.

## NOMENCLATURE

$\mathbf{f}(t)$	forcing vector
$\mathbf{M}$ , $\mathbf{C}$ and $\mathbf{K}$	mass, damping and stiffness matrices respectively
$\mathbf{q}(t)$	response vector
$n$	number of degrees of freedom
FRF	Frequency Response Function
SFEM	Stochastic Finite Element Method