Robust Design Optimization for Crashworthiness of Vehicle Side Impact

Optimization for crashworthiness is of vast importance in automobile industry. Recent advancement in computational prowess has enabled researchers and design engineers to address vehicle crashworthiness, resulting in reduction of cost and time for new product development. However, a deterministic optimum design often resides at the boundary of failure domain, leaving little or no room for modeling imperfections, parameter uncertainties, and/or human error. In this study, an operational model-based robust design optimization (RDO) scheme has been developed for designing crashworthiness of vehicle against side impact. Within this framework, differential evolution algorithm (DEA) has been coupled with polynomial correlated function expansion (PCFE). An adaptive framework for determining the optimum basis order in PCFE has also been presented. It is argued that the coupled DEA–PCFE is more efficient and accurate, as compared to conventional techniques. For RDO of vehicle against side impact, minimization of the weight and lower rib deflection of the vehicle are considered to be the primary design objectives. Case studies by providing various emphases on the two objectives have also been performed. For all the cases, DEA–PCFE is found to yield highly accurate results.

1 Introduction

Investigating the crashworthiness of vehicle to ensure occupant safety and structural integrity is one of the main focuses of automobile industries [1]. Over the last decade, the exponential increase in computational prowess has favored the development of high-fidelity finite element analysis (FEA)-based optimization techniques to tackle industrial problems such as crashworthiness of vehicle [2–9]. However, optimum design often resides in the boundary of system failure, leaving very little or no room for modeling imperfections [10–12], parameter uncertainties [12,13], and/or human error. To overcome this issue, it is necessary to consider the effect of uncertainty in the optimization process. In literature, there exist two distinct approaches, namely robust design optimization (RDO) [14–16] and reliability-based design optimization (RBDO) [17–19], that incorporate uncertainty into the framework of optimization. While the former minimizes the propagation of uncertainty from input to the output, the latter seeks an explicit expression for objective function is not available, the gradient of objective function is calculated by employing finite difference method. This renders the optimization process computationally expensive. Second, the process of uncertainty quantification is computationally intensive and often requires a tradeoff between efficiency and accuracy. Even the popular methods for uncertainty quantification such as perturbation method [27,28], Kriging [29–34], polynomial chaos expansion (PCE) [35–42], moving least square method [43–46], collocation-based approaches [47], tensor product-based approach [48], and radial basis function [49–53] often yields erroneous results when coupled into the framework of RDO. In this context, there is a need of efficient uncertainty quantification tool that can be utilized for solving RDO problems.

Recently, a new operational model, referred to as polynomial correlated function expansion (PCFE) [54–62], has been proposed for uncertainty quantification. As already demonstrated, PCFE yields highly accurate results [54–60]. In this paper, a coupled framework that couples PCFE into the framework of differential evolution algorithm (DEA) [63,64] has been utilized for RDO of vehicle due to side impact. It is argued that the coupled DEA–PCFE is efficient as well as accurate due to the following factors:

• DEA is a global optimization tool and does not yield the local minima. Moreover, it has already been established in previous studies that DEA has a rapid convergence rate [63–66].

• DEA is a gradient-free optimization technique. Therefore, it is equally applicable to both differentiable and nondifferentiable functions.

• PCFE is an efficient uncertainty quantification tool capable of dealing with high-dimensional problems. Thus, using PCFE to reconstruct the objective function and constraints makes the procedure efficient.

The primary focus of this work is to investigate the applicability of DEA–PCFE for solving the crashworthiness of a vehicle due to side impact. The novelty of this work is mainly threefold:

• PCFE is a novel operational model and has never been utilized for solving RDO problems. This is the first instance where PCFE has been utilized in RDO.

• All the existing works carried out on PCFE selects the order of basis empirically. In this paper, a guideline for determining the optimum basis order has been proposed.

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• The newly proposed method is applied to a finite-element model with 1.7 million (M) nodes. Neither DEA nor PCFE have been applied to such a large-scale real-life problem. This application will give further confidence to the readers/users of this method, which has not been reported before.

The rest of the paper is organized as follows. In Sec. 2, a brief description of RDO has been provided. The concepts of DEA–PCFE are briefly reviewed in Sec. 3. In Sec. 4, results obtained using DEA–PCFE have been validated against five popular RDO techniques. The problem description along with results obtained has been discussed in Sec. 5. Finally, Sec. 6 provides the concluding remarks.

2 Fundamentals of Robust Design Optimization

Unlike conventional deterministic design optimization which do not account for uncertainties, a RDO problem develops a solution that is least sensitive to variations of the nominal design. Thus, the goal of RDO is to minimize the effects of uncertainties involved in system design without suppressing their causes [15]. To further illustrate the methodology, Fig. 1 has been presented.

Several past works [15,67–70] have assigned different types of interpretations to RDO depending on the application and uncertainty quantification and hence attracted various forms of problem formulation. A summary of the same has been provided below:

1. Minimization of the mean of objective function while constraining to a predefined level as illustrated by the below equation

\[
\begin{align*}
\min & \mu(f(x)) \\
\text{s.t.} & \ \ x_L \leq x \leq x_U
\end{align*}
\]  

where \( \mu(f(x)) \) denotes the mean value of objective function and \( x_L \) and \( x_U \) represent the lower and upper bounds of the design variable \( x \). Here, the objective function represents a general loss in performance, and hence, the expected value of the same can be considered as a risk that is at par with Bayesian approach in statistical decision theory [71].

2. Minimization of the standard deviation (SD) of objective function pertaining to usual constraints as illustrated by the below equation

\[
\begin{align*}
\min & \sigma(f(x)) \\
\text{s.t.} & \ \ x_L \leq x \leq x_U
\end{align*}
\]  

where \( \sigma(f(x)) \) denotes the standard deviation of objective function and \( x_L \) and \( x_U \) represent the lower and upper bounds of the design variable \( x \). This design leads to robustness in a strict sense with minimum sensitivity with respect to the variation of the probabilistic parameters [72].

3. Multiobjective optimization by minimizing the objectives mentioned in Eqs. (1) and (2) as illustrated by the below equation [15,73]

\[
\begin{align*}
\min & \mu(f(x)), \sigma(f(x)) \\
\text{s.t.} & \ \ x_L \leq x \leq x_U
\end{align*}
\]  

However, concurrent optimization of mean and standard deviation has been observed to conflict with each other. In order to overcome this problem, the objective functions can be combined into a single robust objective function assigning relevant weightages to each of the respective functions [68] as

\[
\begin{align*}
\min & \beta \mu(f(x)) + (1 - \beta) \frac{\sigma(f(x))}{\sigma^*} \\
\text{s.t.} & \ \ x_L \leq x \leq x_U
\end{align*}
\]  

where the weighing factor \( \beta \) represents the relative importance of the two objectives. \( \mu^* \) and \( \sigma^* \) are the outcomes of ideal design. In this study, objective function of the form described in Eq. (4) has been considered.

3 Robust Design Optimization Using Differential Evolution Algorithm–Polynomial Correlated Function Expansion

In this section, DEA and PCFE have been reviewed. Pseudo-code for PCFE has also been provided. Toward the end of this section, a flowchart depicting the coupled framework has been provided.

3.1 Differential Evolution Algorithm. DEA [63,64] is a stochastic direct search method that optimizes a problem by iteratively trying to improve a candidate solution with respect to a given measure of quality. Unlike GBO, DEA does not use the gradient of the problem and is thus equally applicable to both differentiable and nondifferentiable problems. Second, DEA makes few or no assumptions regarding the problem being optimized and searches very large spaces of candidate solution. Furthermore, unlike conventional optimization tools, it works with a population of solutions (solution vector). Since multiple strings are being processed and updated, it is likely that DEA will yield the global optima.

Differential evolution algorithm utilizes \( n_F \) d-dimensional parameter vectors \( x_{i,G} = 1, 2, \ldots, n_F \) as a population for each generation \( G \). The initial vector population is considered to be uniformly distributed over the entire parameter space. DEA generates new parameter vectors by adding the weighted difference between the two population vectors to a third vector. This operation is known as mutation. In the next step, the trial vector is obtained by mixing the parameter vectors obtained after mutation with the target vector. This step is known as crossover. If the objective function obtained corresponding to the trial vector is smaller compared to the target vector, trial vector replaces the target vector. This step is known as selection. Note that each population vector must serve once as the target vector in order to increase the competitions. Next, different steps of DEA have been described.
3.1.1 Mutation. A mutated vector is generated for each target vector as
\[ v_{G+1} = x_{G} + f \cdot (x_{G} - x_{k}) \]  
(5)
where \( k_{1}, k_{2}, k_{3} \in \{1, 2, \ldots, n_{p}\} \) are random integers that are mutually different. It is further ensured that \( k_{1}, k_{2}, k_{3} \) are different from the running integer \( i \). \( F \) is a real constant that controls the amplification of the differential variation \( (x_{G} - x_{k}) \). For further details, readers are referred to Refs. [63–66].

3.1.2 Crossover. The primary aim of this step is to increase the diversity of the perturbed parameter vectors. The trial vector \( u_{G+1} = (u_{1,G+1}, u_{2,G+1}, \ldots, u_{d,G+1}) \) is formed, where
\[ u_{j,G+1} = \begin{cases} v_{j,G+1} & \text{if } \text{rand}(j) \leq c_{k} \text{ or } j = \text{rand}(i) \\ x_{j,G} & \text{if } \text{rand}(j) > c_{k} \text{ and } j \neq \text{rand}(i) \end{cases} \]
(6)
for \( j = 1, 2, \ldots, d \).

In Eq. (6), \( \text{rand}(j) \) is the \( j \)th uniform random number and \( \text{rand}(i) \) is the randomly chosen index. \( c_{k} \) is the crossover parameter and resides in \([0, 1] \). For further details, readers may refer [63–66].

3.1.3 Selection. The final step of DEA is the selection. This step decides the suitability of trial vector. In this step, the trial vector \( u_{G+1} \) is compared to the target vector \( x_{G} \). If the value of objective function corresponding to \( u_{G+1} \) is lower compared to that obtained using \( x_{G} \), then \( u_{G+1} \) is set to be \( u_{G+1} \). On the contrary, if the value of objective function corresponding to \( u_{G+1} \) is greater compared to that obtained using \( x_{G} \), the old value of \( x_{G} \) is retained.

One problem associated with any optimization algorithm is the repeated evaluation of objective and constraint functions. However, for almost all real-life problems, the objective and constraint functions are not explicitly known. In fact, expensive finite-element analysis is required for evaluating the objective and constraint functions. Furthermore, for RDO, the number of function evaluation is even more (for quantifying uncertainties).

In order to bypass this issue, a proposed approach utilizes PCFE. In Sec. 3.2, a brief description of PCFE has been provided.

3.2 Polynomial Correlated Function Expansion. PCFE [54,58,59,74], also known as generalized HDMR [62], is a general set of quantitative model assessment and analysis tool for capturing the high-dimensional relationship between sets of input and output model variables. It can be viewed as an extension of classical functional analysis of variance decomposition [75–77], where component functions are represented by utilizing the extended bases [59,62]. The unknown coefficients associated with the bases are determined by employing a homotopy algorithm (HA) [74,78,79]. HA determines the unknown coefficients by minimizing the least squared error and an objective function. The objective function defines an additional criterion that is enforced on the solution. In PCFE, the hierarchical orthogonality of the component functions is considered to be an additional criterion.

Let \( x = \{x_{1}, x_{2}, \ldots, x_{N}\} \) be a \( N \)-dimensional vector, representing the input variables of a structural system. It is quite logical to express the output \( g(x) \) as a finite series [77]
\[ g(x) = g_{0} + \sum_{k=1}^{N} \sum_{i_{1}<i_{2}<\ldots<i_{k}} g_{i_{1}i_{2}\ldots i_{k}} (x_{1}, x_{2}, \ldots, x_{N}) \]
(7)
where \( g_{0} \) is a constant and represents the mean response. The terms in Eq. (7) reflecting the independent effects only are termed as first-order component function and denoted as \( g_{i}(x_{i}) \). Similarly, the terms reflecting the bivariate effects are termed as second-order component function and denoted by \( g_{i,j}(x_{i}, x_{j}) \). The constant \( g_{0} \) is termed as zeroth-order component function.

Assume two subspace \( R \) and \( B \) in Hilbert space are spanned by basis \( \{r_{1}, r_{2}, \ldots, r_{l}\} \) and \( \{b_{1}, b_{2}, \ldots, b_{k}\} \), respectively. Now if \( (i) \ B \supset R \) and \( (ii) B = R + R^{2} \), where \( R^{2} \) is the orthogonal complement subspace of \( R \), \( B \) is termed as extended basis, and \( R \) is termed as nonextended basis [59,62].

Now if \( \psi \) be some suitable basis for \( x \subseteq X \), where \( X = \{1, 2, \ldots, N\} \), Eq. (7) can be rewritten as [59,62]
\[ g(x) = g_{0} + \sum_{k=1}^{\infty} \left( \sum_{i_{1}=1}^{N-k} \cdots \sum_{i_{k}=1}^{N-k} \sum_{r=1}^{k} \sum_{m_{1}=1}^{\infty} \cdots \sum_{m_{k}=1}^{\infty} \sum_{s_{1}=1}^{\infty} \cdots \sum_{s_{k}=1}^{\infty} \prod_{i=1}^{k} g_{i(i_{1},i_{2},\ldots,i_{k})} \psi_{m_{1}} \psi_{m_{2}} \cdots \psi_{m_{k}} \right) \]
(8)
Noting that most real-life problems exhibit only lower-order cooperative effect, the higher-order component functions can be truncated. Considering up to \( M \)th order component function and \( st \) order basis yields
\[ g(x) = g_{0} + \sum_{k=1}^{M} \left( \sum_{i_{1}=1}^{N-k} \cdots \sum_{i_{k}=1}^{N-k} \sum_{r=1}^{k} \sum_{m_{1}=1}^{\infty} \cdots \sum_{m_{k}=1}^{\infty} \sum_{s_{1}=1}^{\infty} \cdots \sum_{s_{k}=1}^{\infty} \prod_{i=1}^{k} g_{i(i_{1},i_{2},\ldots,i_{k})} \psi_{m_{1}} \psi_{m_{2}} \cdots \psi_{m_{k}} \right) \]
(9)
Once the unknown coefficients associated with the bases are determined, Eq. (9) represents the basic functional form of PCFE.

Remark 1. As already established in previous studies [54,59], most of the real-life problems only involve a lower-order cooperative effect. In fact, results obtained using second-order \((M = 2)\) PCFE are accurate for real-life problems [54,59,59]. Based on this observation, a second-order PCFE has been utilized in this work.

An essential condition associated with Eq. (9) is the hierarchical orthogonality of the component functions [59,62], which requires a higher-order component function to be orthogonal with all the lower-order component functions. To determine the unknown coefficients \( \alpha \) while satisfying the orthogonality criteria, an HA [55,79] is employed. This algorithm treats the orthogonality of component functions as an additional condition and determines the unknown coefficients by minimizing the least-squared error.

Rewriting Eq. (9) in matrix form
\[ \Psi \alpha = d \]
(10)
where \( \Psi \) and \( d \) consists of the basis functions and unknown coefficients, respectively. If \( g = \{g_{1}, g_{2}, \ldots, g_{p}\}^{T} \) be the observed responses at \( q \) sets of sample points, then
\[ d = g - \hat{g} \]
(11)
where \( \hat{g} = \{g_{0}, g_{0}, \ldots, g_{0}\}^{T} \). Premultiplying Eq. (11) by \( \Psi^{T} \), one obtains
\[ \text{B} \alpha = C \]
(12)
where \( \text{B} = \Psi^{T} \Psi \) and \( C = \Psi^{T} d \). From Eq. (9), it is evident that as long as \( N > 1 \), \( \Psi \) have identical columns. Thus, \( \text{B} \) is having identical rows which can be treated as redundants. Removing the redundants
\[ \text{B}' \alpha = C' \]
(13)
where \( \text{B}' \) and \( C' \) are, respectively, \( \text{B} \) and \( C \) after removing the redundants. Equation (13) represents a set of underdetermined
A pseudo-code for determining unknown coefficients of PCFE is shown in Algorithm 1.

### Algorithm 1 Pseudo-code for PCFE

1. Initialize: Provide order of PCFE, input variable (var) type and corresponding parameters. Identify variable bounds based on the input parameters. Generate sample points (\(N_1\)) within the bounds and obtain response \(g(x)\) by actual finite element model analysis.

2. \((\text{var})_0 = \frac{(\text{var}) - \min(\text{var})}{\max(\text{var}) - \min(\text{var})}\) \(\forall i, j\)

3. \(s_0 = \frac{1}{N_1} \sum g(x_i)\)

4. for \(i = 1 : N_1\)
    - \(d_i = g(x_i) - s_0\)
  end for

5. \(\Psi = [\psi(x^1) \quad \psi(x^2) \quad \cdots \quad \psi(x^{N_1})]^T\) where

\[
\psi(x^i) = \begin{bmatrix}
\psi_1(x^i) & \psi_2(x^i) & \cdots & \psi_1(x^i) \\
\psi_2(x^i) & \psi_2(x^i) & \cdots & \psi_2(x^i) \\
\cdots & \cdots & \cdots & \cdots \\
\psi_{N_1}(x^i) & \psi_{N_1}(x^i) & \cdots & \psi_{N_1}(x^i)
\end{bmatrix}
\]

6. \(\mathbf{d} = [d_1 \quad d_2 \cdots \quad d_{N_1}]^T\)

7. \(\mathbf{B} \leftarrow \Psi^T\mathbf{d}\) and \(\mathbf{C} \leftarrow \Psi^T\mathbf{d}^T\)

8. \((\mathbf{B}', \mathbf{d}') \leftarrow \text{remove redundant} (\mathbf{B}, \mathbf{d})\)

9. \(\mathbf{P} \leftarrow I - (\mathbf{B}')^{-1}\mathbf{B}'\)

10. \(\mathbf{W} \leftarrow \text{form weight}(\Psi)\)

11. \([\mathbf{U} \ \mathbf{V}] \leftarrow \text{svd}(\mathbf{PW})\)

12. \(\mathbf{x}_{\text{HA}} = [\mathbf{V}_{q-r}(\mathbf{U}_{q-r}^T \mathbf{V}_{q-r})^{-1}\mathbf{U}_{q-r}^T]z_0\)

### Step 3: Generate sample points

### Step 4: Formulate PCFE-based operational model using Algorithm 1.

### Step 5: If counter = 1, compute the variance of the unknown coefficients (\(\sigma_{\text{counter}}^2\)). Generate additional sample points. Else compute the change in variance of the unknown coefficients \(\Delta \sigma_{\text{counter}}^2 = \sigma_{\text{counter}}^2 - \sigma_{\text{counter} - 1}^2\). If \(\Delta \sigma_{\text{counter}}^2 > \epsilon_1\), generate additional sample points, set counter = counter + 1 and go to Step 4. Else go to Step 6.

### Step 6: Compute error (\(\epsilon_{\text{err}}\)) of the generated PCFE-based operational model using leave-one-out statistical test. If \(\epsilon_{\text{err}} > \epsilon_2\), set \(s = s + 1\) and go to Step 3. Else stop.

Once the optimum basis order is determined, explicit expressions for the objective and/or constraint functions are generated using the PCFE-based operational model. After that, DEA is utilized to solve the RDO problem formulated by using the PCFE-based objective and constraint functions. A flow chart for performing RDO using DEA–PCFE is shown in Fig. 2.

DEA–PCFE has several unique properties:

- The number of actual function evaluation using DEA–PCFE is significantly less as compared to other popular techniques available in the literature. This is because the terms in PCFE are arranged based on the degree of cooperativity between a finite numbers of random variables, and the lower-order component function of PCFE inherently incorporates the higher-order nonlinearity. As a result, if the response is highly nonlinear and higher dimensional cooperative effects of multiple random variables are rapidly diminishing, PCFE is highly effective. In this context, it may be noted that most of the real-life problems generally involve lower-order cooperative effect [81]. Furthermore, the elite convergence rate of DEA makes DEA–PCFE an ideal tool for RDO.
- PCFE is a finite series consisting of \(2^N\) component functions, where \(N\) is the number of random variables. Thus, if the component functions are convergent, PCFE provides an exact solution.
- PCFE is a mean-square convergent series because it minimizes the mean square error from truncation after a finite number of cooperative terms.
The proposed approach utilizes PCFE, which is optimum in Fourier sense. This is due to the fact that the unknown coefficients associated with the basis are determined by considering the hierarchical orthogonality of the component functions. The above features classify DEA–PCFE as an ideal tool for RDO.

4 Validation

In this section, DEA–PCFE has been validated against popular approaches available in the literature. To be specific, two benchmark problems have been solved using the proposed approach. For both the problems, the population size and the generation size in DEA are considered to be 50 and 100, respectively. The crossover parameter and the mutation parameter are set to be 0.5 and 0.8, respectively. Second-order PCFE model has been utilized in this study. For all the problems, the optimum order of basis is determined by using the proposed iterative algorithm. The sample points required to formulate the PCFE model are generated by using a Sobol sequence. However, it is worth mentioning that the proposed DEA–PCFE is applicable to both uniformly and nonuniformly distributed sample points.

4.1 A Mathematical Function. In the first problem, RDO of a mathematical function has been performed. This problem is a well-known benchmark problem for RDO techniques and has been previously solved by Refs. [8,82] The optimization problem reads

\[ \min_{d} c_{O}(d) = \frac{\sigma_{d}(y_{0}(X))}{15} \]

subject to

\[ c_{k}(d) = 3\sigma_{d}(y_{1}(X)) - \langle y_{1}(X) \rangle \]

\[ 1 < d_{1}, d_{2} < 10 \] (16)

where the two functions \( y_{0}(X) \) and \( y_{1}(X) \) are given as

\[ y_{0}(X) = (X_{1} - 4)^{3} + (X_{1} - 3)^{4} + (X_{2} - 5)^{2} + 10 \] (17)

and

\[ y_{1}(X) = X_{1} + X_{2} - 6.45 \] (18)

Both the design variables follow Gaussian distribution with a standard deviation of 0.4.

Differential evolution algorithm–polynomial correlated function expansion has been utilized to solve this problem. Table 1 shows the optimum design obtained using the proposed approach. Results obtained have been compared with tensor product quadrature (TPQ), Taylor’s series (TS), Gaussian process (GP), polynomial chaos expansion (PCE) [36], and sparse grid collocation technique (SGC) [83]. It is observed that DEA–PCFE yields the best results as compared to the other techniques for RDO. Furthermore, the total number of function evaluation using DEA–PCFE \( (n_{t} = 76) \) is significantly less as compared to TPQ \( (n_{t} = 162) \), TS \( (n_{t} = 90) \), GP \( (n_{t} = 256) \), third order PCE \( (n_{t} = 160) \), and SGC (level five) \( (n_{t} = 795) \).

4.2 2-Bar Truss. In this example, a 2-bar truss element, as shown in Fig. 3, has been considered [8,82]. The system is having five independent random variables, namely cross-sectional area \( X_{1} \), the horizontal span (half) \( X_{2} \), material density \( X_{3} \), load \( X_{4} \), and tensile strength \( X_{5} \). The details of random variables are provided in Table 2. The design variables are \( d_{1} = E(X_{1}) \) and \( d_{2} = E(X_{2}) \). The objective of this problem is to minimize the second moment properties of mass of the structure, given that limiting stresses in both members are below the material yield stress. Consequently, the RDO problem is formulated as

<table>
<thead>
<tr>
<th>Methods</th>
<th>( d_{1} )</th>
<th>( d_{2} )</th>
<th>( c_{O}(d^{*}) )</th>
<th>( N_{t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPQ(^b)</td>
<td>3.45</td>
<td>5.00</td>
<td>0.086</td>
<td>162</td>
</tr>
<tr>
<td>TS(^b)</td>
<td>3.50</td>
<td>4.99</td>
<td>0.090</td>
<td>90</td>
</tr>
<tr>
<td>GP</td>
<td>3.37</td>
<td>5.00</td>
<td>0.076</td>
<td>256</td>
</tr>
<tr>
<td>PCE</td>
<td>3.33</td>
<td>4.99</td>
<td>0.076</td>
<td>160</td>
</tr>
<tr>
<td>SGC</td>
<td>3.36</td>
<td>5.00</td>
<td>0.076</td>
<td>795</td>
</tr>
<tr>
<td>DEA–PCFE</td>
<td>3.35</td>
<td>4.99</td>
<td>0.076</td>
<td>76</td>
</tr>
</tbody>
</table>

\(^a\)No. of actual simulations.

\(^b\)Obtained from Ref. [8].
Table 2 Properties of random variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>COV</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$d_1$</td>
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<td>Gaussian</td>
</tr>
<tr>
<td>$X_2$</td>
<td>$d_2$</td>
<td>0.02</td>
<td>Gaussian</td>
</tr>
<tr>
<td>$X_3$</td>
<td>10,000</td>
<td>0.2</td>
<td>Beta$^a$</td>
</tr>
<tr>
<td>$X_4$</td>
<td>800</td>
<td>0.25</td>
<td>Gumbel</td>
</tr>
<tr>
<td>$X_5$</td>
<td>1050</td>
<td>0.24</td>
<td>lognormal</td>
</tr>
</tbody>
</table>

$^a$For beta distribution, both parameters are five.

Table 3 Robust design of 2-bar truss

<table>
<thead>
<tr>
<th>Methods</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$c_0(d^*)$</th>
<th>$N_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TPQ$^b$</td>
<td>11.567</td>
<td>0.3767</td>
<td>1.239</td>
<td>7722</td>
</tr>
<tr>
<td>TS$^b$</td>
<td>10.957</td>
<td>0.3770</td>
<td>1.174</td>
<td>640</td>
</tr>
<tr>
<td>GP</td>
<td>12.783</td>
<td>0.3770</td>
<td>1.37</td>
<td>1280</td>
</tr>
<tr>
<td>PCE</td>
<td>12.6268</td>
<td>0.3771</td>
<td>1.80</td>
<td>1536</td>
</tr>
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<td>SGC</td>
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<td>0.3771</td>
<td>1.21</td>
<td>42,075</td>
</tr>
<tr>
<td>DEA-PCFE</td>
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<td>0.3770</td>
<td>1.174</td>
<td>648</td>
</tr>
</tbody>
</table>

$^b$No. of actual simulations.

$^a$Obtained from Ref. [8].

\[
\begin{align*}
\text{min}_{d \in D} c_0(d) &= \beta_1 \frac{E(y_0(X))}{10} + (1 - \beta_1) \frac{\sigma(y_0(X))}{2} \\
\text{s.t.} \ c_1(d) &= 3\sigma(y_1(X)) - E(y_1(X)) \leq 0 \\
\ c_2(d) &= 3\sigma(y_2(X)) - E(y_2(X)) \leq 0 \\
0.2 \text{cm}^2 &\leq d_1 \leq 20 \text{cm}^2, \ 0.1 \text{ m} \leq d_2 \leq 1.6 \text{ m} 
\end{align*}
\]

where $y_0, y_1$, and $y_2$ are, respectively, mass of the structure, stress in member 1, and stress in member 2.

Table 3 shows the RDO results obtained using DEA–PCFE, TPQ, TS, GP, PCE [36], and SGC [83]. It is observed that DEA–PCFE ($c_0(d^*)$ = 1.189, $N_s = 640$) outperforms TPQ ($c_0(d^*)$ = 1.239, $N_s = 7722$), GP ($c_0(d^*)$ = 1.37), and SGC ($c_0(d^*)$ = 1.21, $N_s = 42,075$) both in terms of accuracy and efficiency.

5 Crashworthiness of Vehicle Due to Side Impact

In this section, the detailed problem formulation including the FE model of vehicle and safety regulations has been presented. The problem description for RDO and the results obtained have been presented toward the end of this section.

5.1 Vehicle Model Description. The proposed approach is utilized for RDO of a large-scale vehicle as shown in Fig. 4. RDO of the vehicle against crashworthiness due to side impact has been investigated. The system model includes a full-vehicle FE structure model$^1$ a side impact dummy FE model and a side barrier [9,84]. The vehicle model consists of 1.7M nodes. In the FE simulation, the initial velocity of the barrier is considered to be 49.89 km/h. RBDO of the above-mentioned vehicle is already been carried out by Ref. [9]. In this study, the primary objective is to perform RDO of the vehicle using the proposed approach.

5.2 Safety Regulations. It is essential for the vehicle design to satisfy the internal and regulated side impact requirements. Two primary side impact protection guidelines that are quite popular in the vehicle market are the (i) National Highway Traffic Safety Administration (NHTSA) side impact procedures for the Federal Motor Vehicle Safety Standard (FMVSS) and Canadian Motor Vehicle Safety Standard (CMVSS) and (ii) European Enhanced Vehicle-Safety Committee (EEVC) side impact procedures. The EEVC has developed four performance top safety rating criteria: Abdomen load, Rib deflection, VC (m/s), and Pubic symphysis force (kN). Table 4 Safety criteria as per European Enhanced Vehicle-Safety Committee

<table>
<thead>
<tr>
<th>Performance</th>
<th>Top safety rating criteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abdomen load</td>
<td>Upper $\leq 1$</td>
</tr>
<tr>
<td>Rib deflection</td>
<td>Middle $\leq 22$</td>
</tr>
<tr>
<td>VC (m/s)</td>
<td>Lower $\leq 0.32$</td>
</tr>
<tr>
<td>Pubic symphysis force (kN)</td>
<td>$\leq 3$</td>
</tr>
</tbody>
</table>

$^1$Obtained from http://www.ncac.gwu.edu/archives/model/index.html
W
r
side impact has been carried out. From practical point of view.

Table 5 Statistical parameters of random parameters in Eq. (20)

<table>
<thead>
<tr>
<th>Random variables</th>
<th>Std. dev.</th>
<th>Type</th>
<th>d(^2)</th>
<th>x</th>
<th>d(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B-pillar inner (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.500</td>
<td>1.000</td>
<td>1.500</td>
</tr>
<tr>
<td>B-pillar reinforce (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.450</td>
<td>1.000</td>
<td>1.350</td>
</tr>
<tr>
<td>Floor side inner (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.500</td>
<td>1.000</td>
<td>1.500</td>
</tr>
<tr>
<td>Cross member</td>
<td>0.100</td>
<td>Normal</td>
<td>0.500</td>
<td>1.000</td>
<td>1.500</td>
</tr>
<tr>
<td>Door beam (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.875</td>
<td>2.000</td>
<td>2.625</td>
</tr>
<tr>
<td>Door belt line (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.400</td>
<td>1.000</td>
<td>1.200</td>
</tr>
<tr>
<td>Roof rail (mm)</td>
<td>0.100</td>
<td>Normal</td>
<td>0.400</td>
<td>1.000</td>
<td>1.200</td>
</tr>
<tr>
<td>Mat. B-pillar inner (GPa)</td>
<td>0.004</td>
<td>Normal</td>
<td>0.192</td>
<td>0.300</td>
<td>0.345</td>
</tr>
<tr>
<td>Mat. floor side inner (GPa)</td>
<td>0.004</td>
<td>Normal</td>
<td>0.192</td>
<td>0.300</td>
<td>0.345</td>
</tr>
<tr>
<td>Barrier height (mm)</td>
<td>10</td>
<td>Normal</td>
<td>Not design variable</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Barrier hitting (mm)</td>
<td>10</td>
<td>Normal</td>
<td>Not design variable</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The RDO problem for crashworthiness of vehicle due to side impact is formulated as

subject to \(k_{\Delta L} + k_{\Delta L} = 1\) kN

where \(W\) is the mass of the vehicle and \(H\) is the lower rib deflection. PSF, B = PILLAR and \(V_{FD}\), respectively, denote pubic symphysis force (PSF), velocity of B-pillar at midpoint and velocity of front door at B-pillar. \(k\) in Eq. (20) denotes the sigma level. Based on users' requirements, its value is to be chosen appropriately. \(\mu\) and \(\sigma\) denote mean and standard deviation, respectively.

The system is having eleven random variables, which include thickness \((d_1 - d_6)\), material properties of critical part \((d_7, d_9)\), barrier height, and hitting position. The first nine random parameters are considered to be design variables. Barrier height and hitting position may vary from \(-30\) mm to \(30\) mm [9]. All the random variables are assumed to be normally distributed. Details of the random variables are given in Table 5.

5.3 Robust Crashworthiness Design Optimization for Side Impact. The RDO problem for crashworthiness of vehicle due to side impact is formulated as

\[
\text{minimize} \quad \frac{W}{W_0} + (1 - \beta) \frac{\mu_H}{\sigma_H}
\]

subject to \(k_{\Delta L} + k_{\Delta L} = 1\) kN

\[
\mu_{(U/M/L)} + k_{\Delta L} = 0.32 \text{ m/s}
\]

\[
\mu_{(U/M/L)} + k_{\Delta L} = 32 \text{ mm}
\]

where \(W\) is the mass of the vehicle and \(H\) is the lower rib deflection. PSF, B = PILLAR and \(V_{FD}\), respectively, denote pubic symphysis force (PSF), velocity of B-pillar at midpoint and velocity of front door at B-pillar. \(k\) in Eq. (20) denotes the sigma level. Based on users' requirements, its value is to be chosen appropriately. \(\mu\) and \(\sigma\) denote mean and standard deviation, respectively.

The system is having eleven random variables, which include thickness \((d_1 - d_6)\), material properties of critical part \((d_7, d_9)\), barrier height, and hitting position. The first nine random parameters are considered to be design variables. Barrier height and hitting position may vary from \(-30\) mm to \(30\) mm [9]. All the random variables are assumed to be normally distributed. Details of the random variables are given in Table 5.

5.4 Results and Discussion. Using the information provided in the Secs. 5.1–5.3, RDO for crashworthiness of vehicle due to side impact has been carried out. From practical point of view, \(k = 3\) in Eq. (20) has been considered. The training points required for PCFE and PCE have been generated using a Sobol's sampling scheme [85,86].

Table 6 Comparison of RDO results obtained using various methods

<table>
<thead>
<tr>
<th>(\beta)</th>
<th>PCE</th>
<th>SGC</th>
<th>DEA–PCFE</th>
<th>FE</th>
<th>PCE</th>
<th>SGC</th>
<th>DEA–PCFE</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0.50</td>
<td>0.30</td>
<td>0.40</td>
<td>0.60</td>
<td>0.50</td>
<td>0.30</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>0.10</td>
<td>0.65</td>
<td>0.45</td>
<td>0.55</td>
<td>0.75</td>
<td>0.65</td>
<td>0.45</td>
<td>0.55</td>
<td>0.75</td>
</tr>
<tr>
<td>0.20</td>
<td>0.80</td>
<td>0.60</td>
<td>0.70</td>
<td>0.85</td>
<td>0.80</td>
<td>0.60</td>
<td>0.70</td>
<td>0.85</td>
</tr>
<tr>
<td>0.30</td>
<td>0.95</td>
<td>0.75</td>
<td>0.85</td>
<td>0.95</td>
<td>0.95</td>
<td>0.75</td>
<td>0.85</td>
<td>0.95</td>
</tr>
<tr>
<td>0.40</td>
<td>1.10</td>
<td>0.90</td>
<td>0.90</td>
<td>1.00</td>
<td>1.10</td>
<td>0.90</td>
<td>0.90</td>
<td>1.00</td>
</tr>
<tr>
<td>0.50</td>
<td>1.25</td>
<td>1.10</td>
<td>1.10</td>
<td>1.20</td>
<td>1.25</td>
<td>1.10</td>
<td>1.10</td>
<td>1.20</td>
</tr>
<tr>
<td>0.60</td>
<td>1.40</td>
<td>1.30</td>
<td>1.30</td>
<td>1.40</td>
<td>1.40</td>
<td>1.30</td>
<td>1.30</td>
<td>1.40</td>
</tr>
<tr>
<td>0.70</td>
<td>1.55</td>
<td>1.50</td>
<td>1.50</td>
<td>1.60</td>
<td>1.55</td>
<td>1.50</td>
<td>1.50</td>
<td>1.60</td>
</tr>
<tr>
<td>0.80</td>
<td>1.70</td>
<td>1.70</td>
<td>1.70</td>
<td>1.80</td>
<td>1.70</td>
<td>1.70</td>
<td>1.70</td>
<td>1.80</td>
</tr>
<tr>
<td>0.90</td>
<td>1.90</td>
<td>1.90</td>
<td>1.90</td>
<td>2.00</td>
<td>1.90</td>
<td>1.90</td>
<td>1.90</td>
<td>2.00</td>
</tr>
<tr>
<td>1.00</td>
<td>2.10</td>
<td>2.10</td>
<td>2.10</td>
<td>2.20</td>
<td>2.10</td>
<td>2.10</td>
<td>2.10</td>
<td>2.20</td>
</tr>
</tbody>
</table>

Table 7 Design variables at optimum point obtained using DEA–PCFE

<table>
<thead>
<tr>
<th>Variables</th>
<th>PCE</th>
<th>SGC</th>
<th>DEA–PCFE</th>
<th>FE</th>
<th>PCE</th>
<th>SGC</th>
<th>DEA–PCFE</th>
<th>FE</th>
</tr>
</thead>
<tbody>
<tr>
<td>(d_1)</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td>(d_2)</td>
<td>0.69</td>
<td>0.70</td>
<td>0.72</td>
<td>0.71</td>
<td>0.72</td>
<td>0.73</td>
<td>0.72</td>
<td>0.72</td>
</tr>
<tr>
<td>(d_3)</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>(d_4)</td>
<td>1.27</td>
<td>1.25</td>
<td>1.32</td>
<td>1.31</td>
<td>0.88</td>
<td>0.81</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>(d_5)</td>
<td>0.68</td>
<td>0.70</td>
<td>0.62</td>
<td>0.62</td>
<td>0.65</td>
<td>0.63</td>
<td>0.66</td>
<td>0.66</td>
</tr>
<tr>
<td>(d_6)</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>(d_7)</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
<td>1.20</td>
</tr>
<tr>
<td>(d_8)</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
<td>0.22</td>
</tr>
<tr>
<td>(d_9)</td>
<td>0.28</td>
<td>0.29</td>
<td>0.29</td>
<td>0.29</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
<td>0.34</td>
</tr>
</tbody>
</table>

6 Conclusion

A novel approach, referred to as DEA based on PCFE, is presented for robust design optimization of vehicle crashworthiness subjected to side impact. DEA–PCFE is formulated by coupled PCFE with DEA. While PCFE is utilized to generate an approximate surrogate model for the uncertain objective functions and/or constraints, DEA is utilized to perform the optimization. The order of basis involved in PCFE is determined by employing an adaptive scheme. The proposed approach reduces the actual function evaluation significantly and is thus suitable for large-scale problems. DEA–PCFE has been validated against popular methods available in literature.

A full-scale finite element model\(^2\), has been optimized. The problem considered is having nine design variables and eleven random variables. Various case studies by providing various emphasis on the mean and deviation components have also been performed. For all the cases, results obtained using DEA–PCFE are in excellent agreement with FE simulations. Furthermore, DEA–PCFE is highly efficient as compared to available techniques.

\(^2\)Obtained from http://www.ncac.gwu.edu/archives/model/index.html
Biswa, S., Kundu, S., and Das, S., 2015, “Inducing Niching Behavior in Differ-
Hooker, G., 2007, “Generalized Functional ANOVA Diagnostics for High-
Chakraborty, S., and Chowdhury, R., 2017, “Polynomial Correlated Function
Chakraborty, S., and Chowdhury, R., 2016, “Assessment of Polynomial Corre-
Chakraborty, S., and Chowdhury, R., 2016, “Modelling Uncertainty in Incom-
Chakraborty, S., and Chowdhury, R., 2016, “Modelling Uncertainty in Incom-
Hooker, G., 2007, “Generalized Functional ANOVA Diagnostics for High-

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[54] Chakraborty, S., and Chowdhury, R., 2016, “Assessment of Polynomial Corre-
[65] Chakraborty, S., and Chowdhury, R., 2015, “Multivariate Function Approxima-