A discrete particle model and numerical modeling of the failure modes of granular materials

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Abstract

Purpose – To present a discrete particle model for granular materials.

Design/methodology/approach – Starting with kinematical analysis of relative movements of two typical circular grains with different radii in contact, both the relative rolling and the relative sliding motion measurements at contact, including translational and angular velocities (displacements) are defined. Both the rolling and sliding friction tangential forces, and the rolling friction resistance moment, which are constitutively related to corresponding relative motion measurements defined, are formulated and integrated into the framework of dynamic model of the discrete element method.

Findings – Numerical results demonstrate that the importance of rolling friction resistance, including both rolling friction tangential force and rolling friction resistance moment, in correct simulations of physical behavior in particulate systems; and the capability of the proposed model in simulating the different types of failure modes, such as the landslide (shear bands), the compression cracking and the mud avalanching, in granular materials.

Research limitations/implications – Each grain in the particulate system under consideration is assumed to be rigid and circular. Do not account for the effects of plastic deformation at the contact points.

Practical implications – To model the failure phenomena of granular materials in geo-mechanics and geo-technical engineering problems; and to be a component model in a combined discrete-continuum macroscopic approach or a two-scale discrete-continuum micro-macroscopic approach to granular media.

Originality/value – This paper develops a new discrete particle model to describe granular media in several branches of engineering such as soil mechanics, power technologies or sintering processes.

Keywords Particle physics, Mathematical modelling, Numerical analysis, Physical properties of materials

Paper type Research paper

1. Introduction

The discrete element method (DEM) combined with the use of different discrete particle models has become an increasing popular approach for studying the behavior of granular materials. The authors are pleased to acknowledge the support of this work by the National Key Basic Research and Development Program (973 Program) through contract number 2002CB412709 and the National Natural Science Foundation of China through contract/grant numbers 50278012, 10272027.
granular materials such as sand and clay, which consist of packed assemblages of particles with voids at the microscopic level. In addition, DEM, as a numerical technique, can be used to simulate the flow phenomena of granular materials such as mud avalanches, while the continuum model, that is efficient to numerically simulate macroscopic motion of granular assembly as a whole, fails to do so. Continuum models perform exceptionally well at a laboratory level but experience severe difficulties at industrial scale due to complex geometric configurations and time scales. For a typical Lagrangian analysis mesh distortion is near intractable and there are a unique set of challenging problems regarding the evolution of damage and subsequent element deactivation. The DEM approach is based on modeling of the interaction between individual grains. Therefore it is crucial to correctly present quantitative descriptions of constitutive relations between interacting (normal and frictional) forces and relative motions of two particles in contact.

There exist many different models that describe the normal and tangential contact forces in particulate systems. Among them are the pioneering work of Cundall and Strack (1979), with subsequent significant publications including the book of Oda and Iwashita (1999), the models introduced and discussed in Elperin and Golshtein (1997), the papers of Iwashita and Oda (1998), Zhou et al. (1999), Zhang and Whiten (1999) and Feng et al. (2002). Thornton (1991), Vu-Quoc and Zhang (1999) and Vu-Quoc et al. (2001) discussed the mechanisms governing the generation and evolution of contacting forces, particularly the quantitative relations of the reduction in the contact radius to the contacting forces. Vu-Quoc and Zhang (1999) and Vu-Quoc et al. (2001) further developed their models for normal and tangential force – displacement relations for contacting spherical particles, accounting for the effects of plastic deformation at the contact points on variation of the radii of the particles at the contacting points due to plastic flow.

Oda et al. (1982) and Bardet (1994) observed significant effects of particle rolling on the shear strength and consequently on the occurrence and evolution of shear bands in particulate system. Iwashita and Oda (1998) developed a modified distinct element model on the basis of the classical DEM proposed by Cundall by introducing a rolling friction resistance moment at each contact as an additional component mechanism for taking into account the effects of particle rolling, but the model did not distinguish between the rolling and the sliding frictional tangential forces at contact, which should be constitutively related to the tangential components of the relative rolling and the relative sliding motion measurements respectively at the contact.

Feng et al. (2002) pointed out that although the study for developing a model capable of capturing the nature of the friction between two typical grains in contact and therefore capable of simulating the friction properly has been in progress for many years, a universally accepted (rolling) friction model has still not been achieved. They also discussed and stressed the importance of rolling friction resistance in the correct simulation of physical behaviors in particulate systems and took into account both the rolling and the sliding friction forces in their model, but not constitutively related to corresponding rolling and sliding motion quantities and not fully incorporated it into the framework of discrete particle models.

It is remarked that the models proposed by Vu-Quoc and Zhang (1999) and Vu-Quoc et al. (2001) are rather advanced as they were developed on the basis of the continuum theory of elastoplasticity, but it seems to the authors that the practical use of the
models in the framework of DEM may be problematic. Suppose that two particles A and B keep contacting each other over a portion of the load history. As the contacting point at particle A at a particular time instant, as a material point, will no longer contact with particle B in general at the next time instant, it will be difficult to keep the information, i.e. the corrected radii due to plastic deformations and the positions, of all contacting points at particle A, over the loading history.

It should be pointed out that a rational model, which is established on the basis of an adequate description of the mechanisms governing the generation and evolution of the tangential friction forces between two particles at contact, especially an adequate description of the mechanism relating to the rolling, has not been achieved as the physical nature of the friction forces between two moving particles in contact are still not fully understood, particularly from a microscopic view point.

The objective of the present paper is to propose a discrete particle model for granular materials. Each grain in the particulate system under consideration is assumed to be rigid and circular. Starting with kinematical analysis of relative movements of two typical grains in contact, both the relative rolling and sliding motion measurements, including the translational and angular velocities (displacements) are defined. The formulae to calculate both the rolling and sliding friction tangential forces, and the rolling friction resistance moment, which are constitutively related to the defined relative motion measurements respectively, are given. In addition, the viscous normal, sliding and rolling dashpots are respectively introduced into each of component constitutive relations to calculate the normal and the tangential (the sliding and rolling) forces and the rolling friction resistance moment. In addition, the normal compression stiffness coefficient is defined to be a function of the normal “overlap” between two particles in contact. The constitutive relations that take into account both the sliding and rolling resistance forces (resistance moment) are integrated into the framework of the DEM. The numerical results are presented to verify the performance of the proposed model in the simulation of different types of failure modes, such as the landslide (shear bands), the compression cracking and the mud avalanching, in granular materials.

2. Kinematic analysis of moving grains in contact
2.1 Relative sliding and rolling between two particles in contact

To analyze interacting forces attributing to relative motions between two particles in contact, we consider two typical disks A and B with radii \( r_A \) and \( r_B \) respectively, in a two-dimensional assemblage of particles. Assume that these two particles remain in contact at a forward neighborhood \( I_n = [t_n, t_n + \Delta t] \) of time \( t_n \) in the time domain as shown in Figure 1. Let \( X(O_n) \) denote the coordinates of the contacting point \( O_n \), as a spatial point at \( t_n \), in the global Cartesian coordinate system \( X \).

A local Cartesian coordinate system \( x^\alpha \) with its origin at \( X(O_n) \) is defined in such a way that the \( y^\alpha \) axis is chosen to be along the line connecting centers \( A_o, B_o \) of disks A and B with its positive orientation from \( B_o \) to \( A_o \), while the \( x^\alpha \) axis is selected to be along the tangent of the two disks with its positive orientation determined to form a right-hand coordinate system.

Let \( X(O_b) \) denote the coordinates of the contacting point \( O_b \), between the two disks A and B, as a spatial point at \( t \in I_n \), referred to the global Cartesian coordinate system \( X \). The translational and rotational velocities of the coordinate system \( x^\alpha \) at \( t_n \) are then defined as
\[ V_o = \lim_{t \to t_n} \frac{X(O_n) - X(O_n)}{t - t_n}; \quad \Omega_o = \lim_{t \to t_n} \frac{\alpha_t - \alpha_n}{t - t_n} \] (1)

where \( \alpha_n, \alpha_t \) are the inclined angles of the \( x^n \) axis and the \( x^t \) axis to the \( X \) axis, respectively, which are assumed positive counter-clockwise as shown in Figure 1.

Denoting \( V_A^o \) and \( V_B^o \), \( \Omega_A \) and \( \Omega_B \) as the translational and angular velocities of centers \( A_o \) and \( B_o \) at \( t_n \) respectively. The translational velocities \( V_A^c \) and \( V_B^c \) of the contacting points \( A_c \) and \( B_c \) at time \( t_n \), as the material points, at disks A and B referred to the coordinate system \( X \) can be expressed as

\[ V_A^c = V_A^o + \Omega_A \times r_A^{c,n}; \quad V_B^c = V_B^o + \Omega_B \times r_B^{c,n} \] (2)

where

\[ r_A^{c,n} = X(O_n) - X(A_0); \quad r_B^{c,n} = X(O_n) - X(B_0) \] (3)

\[ \Omega_A = \Omega_A e_Z; \quad \Omega_B = \Omega_B e_Z \] (4)

The directions of pseudo-vectors \( \Omega_A \) and \( \Omega_B \) are expressed in terms of the unit vector \( e_Z \) normal to the \( x-y \) plane of the coordinate system \( X \) with the right-hand screw rule. For clarity in expressions, the superscript \( n \) of \( r_A^{c,n} \) and \( r_B^{c,n} \) will be omitted hereafter. Obviously, for the two disks in contact we have

\[ r_B^c = -\frac{r_B}{r_A} r_A^c. \]

It is noted that as a repulsive force between the two disks A and B in contact is considered in the model of forces certain amount of “overlap” of the two disks is assumed to exist. Nevertheless in kinematical analysis of two moving disks in contact the point contact (in the 2D case) between the two disks is assumed as an approximation for simplifying the model of kinematics.

The relative sliding translational velocity \( \Delta V \) and relative rolling angular velocity \( \Delta \Omega \) of two particles A and B at the contacting point \( O_n \) at \( t_n \), can be defined, with the use of equations (2)-(4), as
\[ \Delta V = V_A^c - V_B^c = V_A^o - V_B^o + \left( \Omega_A + \Omega_B \frac{r_B}{r_A} \right) e_z \times r_A^c; \quad (5) \]

\[ \Delta \Omega = \Omega_A - \Omega_B = (\Omega_A - \Omega_B)e_z \quad (6) \]

If \( \Delta V = 0, \Delta \Omega \neq 0 \), pure rolling occurs at the contacting point, whereas if \( \Delta \Omega = 0, \Delta V \neq 0 \) pure sliding occurs even if \( \Omega_A, \Omega_B \neq 0 \), provided \( \Omega_A = \Omega_B \). In the case of pure sliding, equations (5)-(6) lead to

\[ \Omega_A = \Omega_B, \quad \Delta V = V_A^o - V_B^o + \Omega_A \left( 1 + \frac{r_B}{r_A} \right) e_z \times r_A^c \quad (7) \]

while in the case of pure rolling it has

\[ \Omega_A \neq \Omega_B, \quad V_B^o = V_A^o + \left( \Omega_A + \Omega_B \frac{r_B}{r_A} \right) e_z \times r_A^c \quad (8) \]

It is remarked from equations (5) and (6) that if only the translational motion of the two particles exists, i.e. \( V_A^o \neq 0 \) and/or \( V_B^o \neq 0 \), and \( \Omega_A = \Omega_B = 0 \), a possible relative movement between the two particles in contact is only pure sliding; however, when the two particles only rotate, i.e. \( \Omega_A \neq 0 \) and/or \( \Omega_B \neq 0 \), and \( V_A^o = 0, V_B^o = 0 \), possible relative movements, in general, may include not only pure rolling, but also both sliding and rolling.

It is noticed that \( V_A^o, V_B^o \) are defined as the time derivatives of the spatial points (i.e. the Lagrangian derivatives) while \( V_o \) is defined as the time derivative of the spatial point (i.e. the Eulerian derivative). As shown in Figure 1 we have \( (r_A^{c,t} = r_A^c) \)

\[ r_A^c + V_o \Delta t = V_A^o + r_A^{c,t} \quad (9) \]

for particle A in which \( V_o \) and \( V_A^o \) are approximately assumed to be constant within the time interval \([t_n, t]\). There is a similar equation for particle B. It is noted that \( O_n, O_t \) are not the same material point, and \( r_A^{c,t}, r_A^c \) shown in Figure 1 are also not the same material line segment vector. As the velocity quantities at time \( t_n \) are considered from Equation (9) and the similar equation

\[ r_B^c + V_o \Delta t = V_B^o + r_B^{c,t} \]

for particle B with

\[ \lim_{t \rightarrow t_n} (r_A^{c,t} - r_A^c) / \Delta t = \Omega_o \times r_A^c \quad \text{and} \quad \lim_{t \rightarrow t_n} (r_B^{c,t} - r_B^c) / \Delta t = \Omega_o \times r_B^c. \]

\( V_A^o, V_B^o \) can be expressed in terms of \( V_o, \Omega_o \) of the local coordinate system \( x^n \) as

\[ V_A^o = V_o - \Omega_o \times r_A^c; \quad V_B^o = V_o - \Omega_o \times r_B^c \quad (10) \]

Substitution of equation (10) into equation (2) gives

\[ V_A^c - V_o = (\Omega_A - \Omega_o) \times r_A^c; \quad V_B^c - V_o = (\Omega_B - \Omega_o) \times r_B^c \quad (11) \]

where \( V_A^c - V_o \) and \( V_B^c - V_o \) represent respectively the translational velocities of the contacting points at particles A and B that contact each other at spatial point \( X(O_n) \) at \( t_n \), relating to the local coordinate system \( x^n \).
2.2 Relative movement measurements between two particles in contact

As the relative movement measurements within an incremental time step $[t_n, t_{n+1}]$ are concerned, the forward difference scheme in time is assumed to be employed. With the use of equations (5) and (11), the relative sliding displacement increment vector $\Delta U_s$ between two particles A and B in contact that occurs within the time interval can be defined as

$$\Delta U_s = \Delta V dt = (V_A^c - V_B^c) \Delta t = (V_A^c - V_o) \Delta t - (V_B^c - V_o) \Delta t$$  (12)

By using equation (11) and noting

$$r^c_B = -\frac{r_B}{r_A} r^c_A,$$

the projection of $\Delta U_s$ to the $x^n$ axis of the local coordinate system $x^n$, i.e. the relative tangential sliding displacement increment $\Delta u_s$ between two particles A and B at the contacting point can be expressed by

$$\Delta u_s = r_A(\Omega_A - \Omega_o) \Delta t + r_B(\Omega_B - \Omega_o) \Delta t$$  (13)

By denoting $\Delta \theta_A = \Omega_A \Delta t$, $\Delta \theta_B = \Omega_B \Delta t$, $\Delta \theta_o = \Omega_o \Delta t$ and

$$\Delta a = r_A(\Delta \theta_A - \Delta \theta_o); \quad \Delta b = r_B(\Delta \theta_B - \Delta \theta_o)$$  (14)

we can re-write Equation (13) in the form

$$\Delta u_s = \Delta a + \Delta b$$  (15)

According to Equation (6) the relative rolling angular displacement increment between two particles A and B in the time interval from $t_n$ to $t_{n+1}$ is given by

$$\Delta \theta_r = \Delta \Omega \Delta t = (\Omega_A - \Omega_B) \Delta t = \Delta \theta_A - \Delta \theta_B$$  (16)

It is noticed that the two terms in the last equality of equation (12) represent the relative sliding displacement increments of two particles A and B relating to the contacting point $O_n$ at $X(O_o)$. The relative rolling displacement increments of two particles A and B relating to $O_n$ at $X(O_n)$ can be similarly defined by

$$\Delta U^{Ao}_r = \Delta t(\Omega_A - \Omega_o) \times r^c_A \quad \Delta U^{Bo}_r = \Delta t(\Omega_B - \Omega_o) \times r^c_B$$  (17)

The vector $\Delta U_r$ of the relative rolling displacement increments between two particles A and B at $O_n$ can be expressed in the form

$$\Delta U_r = \Delta U^{Ao}_r + \Delta U^{Bo}_r$$  (18)

The projection of $\Delta U_r$ to the $x^n$ axis of the local coordinate system $x^n$, i.e. the relative tangential rolling displacement increment $\Delta u_r$ between two particles A and B at the contacting point is then expressed by

$$\Delta u_r = \Delta a - \Delta b$$  (19)
3. A computational model for friction resistances between particles in contact

Even though the importance of the rotations of soil particles in simulation of mechanical behaviors of granular materials such as soils and clays is well recognized (Iwashita and Oda, 1998; Oda et al., 1982; Bardet, 1994), the effects of rolling friction are still largely neglected in existing discrete particle models (Elperin and Golstein, 1997; Zhang and Whiten, 1999; Tanaka et al., 2000; Takafumi et al., 1998; Zhang and Whiten, 1998). Feng et al. (2002) pointed out that correct modeling of rolling friction is still an under-developed area and many issues remain unanswered and they developed a rolling resistance model and incorporated it within the sliding friction model.

It should be remarked that as two particles in contact move against each other through sliding and rolling, rolling resistances may be neglected in the model for calculating the tangential force as the rolling friction coefficient is, in general, much smaller than the sliding friction coefficient. However, as the relative motion that occurs at the contacting point is only or almost only the relative rolling, a model for tangential forces with omission of rolling resistance may lead to unrealistic results in numerical calculations.

Based on the previous work (Cundall and Strack, 1979; Iwashita and Oda, 1998; Feng et al., 2002), a model for calculation of tangential forces between two particles at contact, which takes into account both rolling resistances (rolling friction tangential force and rolling resistance moment) and sliding frictional force, is proposed. The model is incorporated into the framework of the DEM and applied to simulate different failure modes of granular materials. Each of the friction resistances applied to two particles at contact, i.e. the rolling friction tangential force $F_{tr}$, the rolling resistance moment $M_{tr}$, the sliding friction tangential force $F_{ts}$, are related to corresponding relative movement measurements $\Delta u_t$, $\Delta \theta_t$, $\Delta u_s$ and their variation rates with respect to time.

3.1 Rolling and sliding friction tangential forces

Hereafter within this section the subscript $\tau = r, s$ represents in turn the rolling and the sliding friction tangential force, respectively.

The predictor $F_{tr}^{n+1}$ of rolling/sliding friction tangential force $F_{tr}$ at $t_{n+1}$ due to the relative tangential rolling/sliding displacement increment $\Delta u_{tr}(dt_{tr})$, which occurs within the typical time sub-interval $[t_n, t_{n+1}]$, can be calculated as

$$F_{tr}^{n+1} = f_{tr}^{n+1} + d_{tr}^{n+1}$$

(20)

in which

$$f_{tr}^{n+1} = f_{tr}^n + \Delta f_{tr}; \quad \Delta f_{tr} = -k_{tr} \Delta u_{tr}; \quad d_{tr}^{n+1} = -c_{tr} \frac{du_{tr}}{dt}$$

(21)

where $k_{tr}, c_{tr}$ stand for the stiffness coefficient and the coefficient of viscous damping of the rolling/sliding tangential friction, $d_{tr}^{n+1}$ reflects the damping effect on rolling/sliding friction tangential force in the dynamic model. $F_{tr}^{n+1}$ has to satisfy Coulomb law of friction and rolling/sliding friction tangential force is determined by

$$F_{tr}^{n+1} = F_{tr}^{n+1} \quad \text{if} \quad \left| F_{tr}^{n+1} \right| \leq \mu_r \left| F_N^{n+1} \right|$$

(22a)
\[ F^{n+1}_r = \text{sign} \left( F^{n+1}_{r,tr} \right) \mu_r |F^{n+1}_N|, \quad \text{if} \quad |F^{n+1}_{r,tr}| > \mu_r |F^{n+1}_N| \]  

(22b)

where \( F^{n+1}_N \) is the normal contact force at \( t_{n+1} \), \( \mu_r \) is the (maximum) static rolling/sliding tangential friction coefficient.

3.2 Rolling friction resistance moment

The predictor \( M^{n+1}_{r,tr} \) of rolling friction resistance moment \( M^{n+1}_r \) at \( t_{n+1} \) due to the relative rolling angular displacement increment \( \Delta \theta_r (d \theta_r) \), which occurs within the time sub-interval \([t_n, t_{n+1}]\), can be calculated as

\[ M^{n+1}_{r,tr} = M^{n+1}_{rs} + M^{n+1}_{rv} \]  

(23)

in which

\[ M^{n+1}_{rs} = M^n_{rs} + \Delta M_{rs}; \quad \Delta M_{rs} = -k_{\theta} \Delta \theta_r; \quad M^{n+1}_{rv} = -c_{\theta} \frac{d \theta_r}{dt} \]  

(24)

where \( k_{\theta}, c_{\theta} \) stand for the stiffness coefficient and the coefficient of viscous damping of the rolling friction moment, \( M^{n+1}_{rv} \) reflects the damping effect on rolling friction resistance moment in the dynamic model. \( M^{n+1}_{r,tr} \) has to satisfy Coulomb law of friction and rolling friction resistance moment is determined by

\[ M^{n+1}_r = M^{n+1}_{r,tr} \quad \text{if} \quad |M^{n+1}_{r,tr}| \leq \mu_r |F^{n+1}_N| \]  

(25a)

\[ M^{n+1}_r = \text{sign} \left( M^{n+1}_{r,tr} \right) \mu_r |F^{n+1}_N| \quad \text{if} \quad |M^{n+1}_{r,tr}| > \mu_r |F^{n+1}_N| \]  

(25b)

where \( \mu_r \) and \( r \) are the (maximum) static rolling friction moment coefficient and the radius of the particle in consideration respectively. In fact \( e = r \mu_r \) represents an eccentricity of the normal contact traction \( F^{n+1}_N \) toward the rolling direction with regard to its stationary position (Feng et al., 2002) in view of the line contact (in the 2D case) between the two disks A and B in contact, which occurs in practice and we consider in the model of forces. Hence, the limit \( \mu_r |F^{n+1}_N| \) of rolling friction resistance moment defined in equation (25a) and (25b) for each of two disks A and B in contact possesses the same value, i.e. \( e = r_A \mu_{\theta_A} = r_B \mu_{\theta_B} \) and it will be reached for the two particles simultaneously. Indeed \( e = r_A \mu_{\theta_A} = r_B \mu_{\theta_B} \) is assumed for each two particles in contact in numerical examples of present work.

3.3 The tangential friction resistance force due to both rolling and sliding

In general, both the sliding and the rolling coexist between two particles in contact. The predictor \( F^{n+1}_{T,tr} \) of the tangential friction force \( F^{n+1}_T \) at \( t_{n+1} \) due to the relative tangential sliding and rolling displacement increments \( \Delta u_s (du_s) \) and \( \Delta u_r (du_r) \), which occur within a typical time sub-interval \([t_n, t_{n+1}]\), can be calculated as

\[ F^{n+1}_{T,tr} = F^{n+1}_{s,tr} + F^{n+1}_r \]  

(26)

\( F^{n+1}_{T,tr} \) has to satisfy Coulomb law of friction and the tangential friction force is determined by
\[ F_T = F_{T,fr} \text{ if } |F_{T,fr}| \leq \mu_s F_N \]  
\[ F_T = \text{sign}(F_{T,fr}) \mu_s F_N \text{ if } |F_{T,fr}| > \mu_s F_N \]

It is noted that since \( \mu_s \geq \mu_r \) in general, we have from equations (22a) and (22b)
\[
\max(|F_T|) = \max(|F_s|) = \mu_s F_N
\]  

For simplifying the discussion below in this section, we assume particle B is fixed and the subscript and the superscript for identifying particle A is omitted for clarity in expressions.

It is emphasized that in a pure rolling case, a rolling frictional force should be applied in the tangential direction (Feng et al., 2002). To clarify this point, consider the case of pure rolling between two particles A and B at contact, i.e. the relative sliding velocity at the contacting point \( v_s^c (= \dot{u}_s) = 0 \) due to \( \Delta V = 0 \) and therefore, \( \mathbf{V}_A^c = 0 \). For particle A we have
\[
v_s^c = v^o + r \dot{\Omega} = 0
\]  

where \( v^o \) is the tangential translation velocity of the center of the particle, \( r, \dot{\Omega} \) are the radius and the rolling angular velocity of the particle. It is obtained from equation (29) that
\[
\dot{v}^o + r \ddot{\Omega} = 0
\]  

On the other hand, Newton’s second law governing the tangential translation and the rolling movements of particle A subjected to a tangential external force \( F_e \) at the particle center can be expressed as
\[
m \dot{v}^o = F_e + F_T
\]
\[
I_m \ddot{\Omega} = F_T r + M_r
\]

where \( m \) and \( I_m \) are the mass and the mass moment of inertia of particle A. As the steady rolling (\( \dot{\Omega} = 0 \)) under the condition \( F_s \neq 0 \) is considered, we have \( \dot{v}^o = 0 \) from equation (30). Since the relative movement between the two particles is pure rolling, i.e. \( F_s = 0 (\dot{u}_s = 0) \), if the rolling friction resistance is omitted, then \( F_T = F_s = 0 \). This obviously violates the motion equation (31). Hence, it is concluded that even though \( \mu_s \geq \mu_r \) in general and \( |F_s| \geq |F_T| \) in most cases when the relative sliding and the relative rolling coexist, to ensure the reliability of the numerical simulations it is necessary to take into account the effects of both the sliding and the rolling friction resistances in the model for calculation of tangential friction resistances. Furthermore, as steady state pure rolling is concerned (\( \dot{\Omega} = 0 \)) and an external force \( F_e \neq 0 \) is assumed to apply at the center of particle A, we must have both a resulting \( F_r = F_T = -F_e \) and \( M_r = -F_T r = -F_e r F_s r \) at the contact point from equations (29)-(32). This conclusion justifies the inclusion of both \( F_r \) and \( M_r \) simultaneously in the proposed model, which provides a mechanism for energy dissipation during steady state rolling.
3.4 Repulsive normal contact force

A repulsive normal force between two particles A and B in contact arises due to the stiffness of the particles. The model for calculating the normal force is based on the assumption that a small amount of “overlap” of the two particles in contact is allowed and compressibility of the particle is finite. In the proposed model, the normal contact force is related to the relative normal movement measurement, i.e. the “overlap” $u_{N}^{n+1}$ at current time instant $t_{n+1}$ and its variation rate with respect to time. $u_{N}^{n+1}$ is defined as the difference between the sum of the radii of the two particles and the distance between the centers of the two particles, i.e.

$$u_{N}^{n+1} = r_{A} + r_{B} - |X(A^{n+1}) - X(B_{o}^{n+1})|$$

(33)

The normal contact force between the two particles in contact can be calculated by

$$F_{N}^{n+1} = -k_{N}u_{N}^{n+1} - c_{N}\frac{u_{N}^{n+1} - u_{N}^{n}}{\Delta t} \quad \text{if} \quad u_{N}^{n+1} > 0$$

(34a)

$$F_{N}^{n+1} = 0 \quad \text{if} \quad u_{N}^{n+1} \leq 0$$

(34b)

where $k_{N}, c_{N}$ are the compression stiffness coefficient and the coefficient of viscous damping of the normal contact deformation for the granular material. In the present model both the non-linear normal compression stiffness coefficients and the linear (i.e. $k_{N}$ is taken as a constant) at contact are employed in the first and the second numerical examples respectively. The non-linear normal compression stiffness coefficient is proposed on the basis of combination of the power law model (Han et al., 2000) and the linear model for the coefficient, i.e.

$$k_{N} = k_{N}(u_{N}^{n+1}) = k_{N0} \exp\left(\frac{u_{N}^{n+1}}{r_{A} + r_{B}}\right)$$

(35)

in which $k_{N0} = k_{N}(0)$ is the initial normal compression stiffness coefficient.

4. The governing equations for dynamic equilibrium of discrete particulate system – a discrete element approach

Let $\Xi_{A}$ and $n_{A}$ denote the set of neighboring particles and the number of these particles for a given typical particle A at time $t \in [t_{n}, t_{n+1}]$. The neighboring particles of particle A are defined as those particles, which have the possibility of keeping contact with particle A at time $t \in [t_{n}, t_{n+1}]$. A subset $\Xi_{A}^{c} \in \Xi_{A}$ is formed after checking each of the particles in the set $\Xi_{A}$. Each of the particles in the subset $\Xi_{A}^{c}$ keeps contacting with particle A at $t \in [t_{n}, t_{n+1}]$. Using the DEM, the motion equations of particle A in the two-dimensional case can be written as

$$m_{A} U_{A}^{n+1} = \sum_{x \in \Xi_{A}^{c}} F_{x}^{A} + F_{x}^{e,A}$$

(36a)
where \( m^A, \dot{I}_m^A \) are, respectively, the mass and the mass moment of inertia of particle A, \( \ddot{U}_X^A, \ddot{U}_Y^A, \ddot{\theta}^A \) stand for the translational accelerations in \( X, Y \) axes and the angular acceleration in the \( X-Y \) plane of particle A. \( F_{x^i}^{e, A}, F_{y^i}^{e, A}, M_{r}^i \) are the external loads applied to particle A corresponding to the degrees of freedom \( U_X^A, U_Y^A, \theta^A \). The rolling friction resistance moment \( M_i^r \) exerted by particle \( i \) on particle A can be calculated by using equation (25a) and (25b). \( F_{X,i}^T = \begin{bmatrix} F_{x^i}^T & F_{y^i}^T \end{bmatrix} \) is the contacting force produced by particle \( i \) on particle A referred to the global coordinate system \( \textbf{X} \). Let \( \alpha_i \) denote the angle between the \( X \) axis of global coordinate system \( \textbf{X} \) to the \( x \) axis of the local coordinate system with its origin at the contacting point between particle A and particle \( i \) as shown in Figure 1, in which the local coordinate system is denoted as \( \textbf{x}^n \).

The contacting force exerted by particle \( i \) on particle A in the local coordinate system \( \textbf{x}^n \) is denoted by \( F_{x,i}^T \). The tangential force \( F_{x^i}^T \) is defined by equation (27a) and (27b) and can be calculated by using equations (20)-(22a), (22b). The transformation between \( F_{x,i}^T \) and \( F_{x,i}^T \) can be expressed in the form

\[
F_{x,i}^T = T_i^T F_{x,i}^T, \quad T_i = \begin{bmatrix} \cos \alpha_i & \sin \alpha_i \\ -\sin \alpha_i & \cos \alpha_i \end{bmatrix}
\] (37)

It is noted that the right hand side terms of equation (36a), (36b) and (36c) are not only determined by the external loads locally applied to particle A, but also related to the relative movements of particle A with its neighboring points in contact, i.e. \( U_X^i, U_Y^i, \theta^i, U_X^i, U_Y^i, \theta^i \ (i \in \Xi_A) \) and their derivatives with respect to time.

An explicit time integration algorithm is used to solve the motion equations (36a), (36b) and (36c) for each particle in the particulate system under consideration. The conditional stable nature of explicit algorithms in time domain imposes a limitation on the maximum time step that can be employed in the solution procedure. A formula for calculating critical time step size is given below (Tanaka et al., 2000)

\[
\Delta t_{cr} = \lambda_{cr} \sqrt{m/k_n}
\]

where \( k_n \) and \( m \) are the (equivalent) normal contact stiffness coefficient and the (equivalent) mass of a typical particle, \( \lambda_{cr} \) is the coefficient to take into account the viscous damping effect and \( \lambda_{cr} = 0.75 \) is chosen in (Tanaka et al., 2000). In the present work, the critical time step size for the whole of the particulate system is calculated according to the following formula

\[
\Delta t_{cr} = \lambda_{cr} \min_i \sqrt{m_i/k_{n,i}}
\] (38)
5. Nominal strains of granular materials

To measure the change of the position of a particle in consideration relating to its neighboring particles two nominal strains, termed the effective strain and the volumetric strain, respectively, are defined at the center of the particle.

Consider the change of the position of particle A relating to one of its neighboring particles denoted particle B, as shown in Figure 2. Let $X^n_A$, $X^n_B$, $X^{n+1}_A$, and $X^{n+1}_B$ denote the coordinates of the centers of particles A and B, referred to the global coordinate system $X$ at two time instants $t_n$ and $t_{n+1}$. The relative positions of particles A and B referred to the global coordinate system at $t_n$ and $t_{n+1}$ can be expressed as $\Delta X^n_{BA} = X^n_B - X^n_A$ and $\Delta X^{n+1}_{BA} = X^{n+1}_B - X^{n+1}_A$, respectively. Referred to the local coordinate system $(x^n_A, y^n_A)$ defined in terms of the positions of particles A and B at time $t_n$ as shown in Figure 2, the relative positions of particles A and B at $t_n$ and $t_{n+1}$ can also be expressed as $\Delta x^n_{BA} = x^n_B - x^n_A$ and $\Delta x^{n+1}_{BA} = x^{n+1}_B - x^{n+1}_A$, where $x^n_A$, $x^n_B$, and $x^{n+1}_A$, $x^{n+1}_B$ are the local coordinates of the centers of particles A and B at $t_n$ and $t_{n+1}$. According to the coordinate transformation, we have

$$
\Delta x^n_{BA} = T \Delta X^n_{BA}, \quad \Delta x^{n+1}_{BA} = T \Delta X^{n+1}_{BA}, \quad T = \begin{bmatrix} \cos \alpha_1 & \sin \alpha_1 \\ -\sin \alpha_1 & \cos \alpha_1 \end{bmatrix}
$$

The change in the relative position of the pair of material particles A and B from time $t_n$ to $t_{n+1}$ can be described by the following deformation gradient $f_n$ referred to the local coordinates

$$
f_n = \frac{\Delta x^{n+1}_{BA}}{\Delta x^n_{BA}} = R_n U_n
$$

where

$$
R_n = \begin{bmatrix} \cos(\alpha_2 - \alpha_1) & -\sin(\alpha_2 - \alpha_1) \\ \sin(\alpha_2 - \alpha_1) & \cos(\alpha_2 - \alpha_1) \end{bmatrix}, \quad U_n = \begin{bmatrix} \lambda_{AB} & 0 \\ 0 & 1 \end{bmatrix}
$$

![Figure 2](image_url)

Change of the position of particle A relating a neighboring particle.
\[ \lambda_{AB} = \frac{r_{AB}^{n+1}}{r_{AB}^n}, \quad p_{AB}^n = |\Delta x_{BA}^n|, \quad p_{AB}^{n+1} = |\Delta x_{BA}^{n+1}| \]  

(42)

in which \( \alpha_1 \) and \( \alpha_2 \) denote the angles between the \( X \)-axis of the global coordinate system \( X \) to the \( x \) axis of the local coordinate systems at the two time instants, as shown in Figure 2. Substitution of equation (39) into equation (40) gives

\[ \Delta x_{BA}^{n+1} = F \Delta x_{BA}^n \]  

(43)

in which

\[ F = T^T f_n T \]  

(44)

Further we denote the “displacement derivative” matrix

\[ D = F - I \]  

(45)

where \( I \) is the identity matrix, and

\[ \gamma_{AB} = \left[ \frac{2}{3} D_{ij} D_{ij} \right]^{1/2} \]  

(46)

with components \( D_{ij} \) of matrix \( D \). The effective strain at the center of particle \( A \) measured in terms of the changes in the relative positions of particle \( A \) with its \( n_A \) neighboring particles can be defined, according to the theory of continuum mechanics, as

\[ \gamma_A = \frac{1}{n_A} \sum_{B=1}^{n_A} \gamma_{AB} \]  

(47)

The volumetric strain at the center of particle \( A \) is then defined as

\[ \gamma_A^v = \frac{1}{n_A} \sum_{B=1}^{n_A} \gamma_{AB}^v, \quad \gamma_{AB}^v = \lambda_{AB} - 1 \]  

(48)

6. Numerical example

6.1 Uni-axial compression of a rectangular panel

The first example concerns a granular assembly with 86.7 \( \times \) 50 cm rectangular profile. The assembly is generated by 4,950 homogeneous particles with radius of 5 mm collocated in a regular manner and subjected to a uniaxial compression between two rigid plates applied by a vertical displacement control, as shown in Figure 3. This granular assembly with radius of 5 mm is characterized by “smaller grains” and termed “S.G..” for short. The gravitational forces are neglected. The particles on the left and the right boundaries are free in the vertical and the horizontal directions. The contact between the particles on the top and the bottom boundaries with the plates is modeled as a vertical sticking, i.e. only the vertical displacements of the particles on the top and the bottom boundaries are specified as uniformly prescribed values under the vertical displacement control. The horizontal movements of the particles on the top and the bottom boundaries relating to the plates are allowed with the sliding coefficient 0.5 being used. The material parameters of the granular assembly used in the example are listed in Table I.
To demonstrate the importance of the rolling resistance, particularly the rolling friction tangential force, on the shear strength and consequently on the occurrence and evolution of shear bands in particulate system, the numerical results obtained by the cases with and without the rolling resistance accounted into the model are compared. The four cases are particularly considered, i.e.

1. without rolling \( (k_r = 0, k_\theta = 0) \);
2. with rolling A \( (k_r = 0, k_\theta = 2.5 \text{Nm/rad}) \);
3. with rolling B \( (k_r = 10^6 \text{N/m}, k_\theta = 2.5 \text{Nm/rad}) \);
4. with rolling C \( (k_r = 10^8 \text{N/m}, k_\theta = 2.5 \text{Nm/rad}) \).

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<td>Stiffness coefficient of normal force ( (k_n) )</td>
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<tr>
<td>Stiffness coefficient of sliding force ( (k_s) )</td>
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</tr>
<tr>
<td>Stiffness coefficient of rolling force ( (k_r) )</td>
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<tr>
<td>Stiffness coefficient of rolling moment ( (k_q) )</td>
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</tr>
<tr>
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<tr>
<td>Damping coefficient of sliding force ( (c_s) )</td>
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</tr>
<tr>
<td>Damping coefficient of rolling force ( (c_r) )</td>
<td>( 0.4 ) (N s/m)</td>
</tr>
<tr>
<td>Damping coefficient of rolling moment ( (c_q) )</td>
<td>( 0.4 ) (Nm s/rad)</td>
</tr>
<tr>
<td>Sliding friction force coefficient ( (\mu_s) )</td>
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</tr>
<tr>
<td>Rolling friction moment coefficient ( (\mu_q) )</td>
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</tr>
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</table>

Table I. The material parameters of the granular assembly used in the rectangular panel example.
Here effects of the rolling friction tangential force are particularly symbolized by the material datum $k_r$, which does not appear in existing discrete particle models as the rolling and the sliding friction tangential forces at contact were not distinguished in these models.

The load-displacement curves to show the load history applied on the top of the granular assembly with the increasing vertical displacements of top surface of the plate are shown in Figure 4(a). The results illustrate the reduction of the load-carrying capability, i.e. the softening behaviour, of the assembly. Figures 5-7 show evolutions of the effective strain, the volumetric strain and the particle rotation distributions, respectively, with increasing vertical displacements 0.6, 1.2, 1.8 and 3.6 cm prescribed to the particles at the top boundary of the assembly, which show that the effective and the volumetric strains and the particle rotation occur and develop sharply into a narrow band of intense straining characterized strain localization phenomena. It is

**Figure 4.**
The load-displacement curves for the rectangular granular assemblies regularly or randomly generated by particles with different radii subjected to uniaxial uniform compression; (a) for regularly generated particulate assembly; and (b) for randomly generated particulate assembly.
observed from Figures 4-6 that the present discrete particle model is capable of modeling the failure mode of shear bands characterized by the softening behavior and strain localization phenomena.

To show the dependence of the width of the shear band and the load-carrying capability on the particle radius (the particle size), the granular assembly with the same profile is re-generated by 1,225 homogeneous particles with radius of 10 mm collocated in a regular manner. The abbreviated name of the assembly is termed “L.G..” (i.e. larger grains) for short. The material parameters used in this case are chosen as the same as those used for the case with the finer size particles except for $\mu_0 = 0.01$. Figure 8 shows the effective and the volumetric strain distributions as a vertical displacement of 1.8 cm is prescribed to the top boundary of the coarser granular assembly. It is obviously observed in the figure that the shear bands obtained by using the coarser granular assembly are wider than those shown in Figures 5-6 by using the finer granular assembly. The particle size can be regarded to act as the internal length scale in the discrete particle model and is related to the width of the shear band. On the other hand, Figure 4(a) shows that the limit of load-carrying capability of the coarser

Figure 5.
The effective strain distributions in the rectangular granular assembly with increasing vertical displacements prescribed to the top boundary of the assembly ($k_s = 1.0 \times 10^8$ N/m); (a) 0.6 cm; (b) 1.2 cm; (c) 1.8 cm; and (d) 3.6 cm.
granular assembly is lower than half of that of the finer granular assembly. It may be attributed to the less total contacting points, therefore, the smaller total internal friction resisting the external force causing material failure.

It is remarked that the shear band mode and its width depend not only the particle size as shown in Figures 5-6 and Figure 8, but also the material property data, particularly the stiffness coefficient $k_s$ of the sliding tangential friction. Figures 9-10 shows evolutions of the effective strain and the volumetric strain distributions, which occur and develop in the “S.G.” assembly as the material parameters are chosen the same as those listed above except $k_s = 1.0 \times 10^6$ N/m. It is shown in Figures 9-10 that the shear band mode in this case is rather different from that occurs as shown in Figures 5-6, where the four shear bands appear at the final stage of the simulation while only two shear bands develop until the end of the simulation in Figures 9-10, that is similar to the shear band mode developed in the rectangular panel example shown in (Li et al., 2003), in which the example is modeled by using the continuum theory combined with the finite element discretization.

Figure 6.
The volumetric strain distributions in the rectangular granular assembly with increasing vertical displacements prescribed to the top boundary of the assembly ($k_s = 1.0 \times 10^6$ N/m); (a) 0.6 cm; (b) 1.2 cm; (c) 1.8 cm; and (d) 3.6 cm
Figure 7.
The particle rotation distributions in the rectangular granular assembly with increasing vertical displacements prescribed to the top boundary of the assembly ($k_s = 1.0 \times 10^8$ N/m); (a) 0.6 cm; (b) 1.2 cm; (c) 1.8 cm; and (d) 3.6 cm.

Figure 8.
The effective and the volumetric strain distributions as a vertical displacement of 1.8 cm is prescribed to the top boundary of the coarser granular assembly ($k_s = 1.0 \times 10^8$ N/m); (a) effective strain distribution; and (b) volumetric strain distribution.
The mechanical properties of granular materials not only depend on the physical properties and the geometrical sizes of individual particle in the granular assembly, but are also related to the collocation manner of all particles in the assembly. To illustrate this, we consider a granular assembly with 80 $\times$ 60 cm rectangular profile. The assembly is randomly generated by 3,679 particles with different radii of 4.35, 5.8 and 7.25 mm and termed “R.G.” for short. The material parameters used in this case are chosen as the same as those used for the above assemblies generated by homogeneous particles except $\mu_{0, i} r_i = 1 \times 10^{-4}$ m ($i = 1, 2, 3$), where $r_i, \mu_{0, i}$ are the radius and corresponding value of $\mu_0$ for each of the three types of particles. The gravitational forces are neglected. It is assumed that there exist neither initial overlaps (initial contacting forces) between each two particles in contact nor initial microforces undertaken at any particles in the assembly. Figure 11 shows evolutions of the effective strain distribution with increasing vertical displacements 1.2, 12, 30 and 43.2 cm prescribed to the particles at the top boundary of the randomly generated particle assembly.

Figure 9.
The effective strain distributions in the rectangular granular assembly with increasing vertical displacements prescribed to the top boundary of the assembly ($k_s = 1.0 \times 10^6$ N/m); (a) 0.6 cm; (b) 1.2 cm; (c) 1.8 cm; and (d) 3.6 cm.
It should be noted that no obvious shear band characterizing failure mode of strain localization, but severe vertical settlement with dramatic dilation in horizontal direction are observed in the assembly randomly generated by particles with different radii. It can be attributed to the stochastic distribution of voids with different sizes in the assembly, therefore, the particles with smaller radii tend to fill into the pores enclosed by the particles with larger radii under vertical compression; and the stochastic anisotropy due to random collocation manner of particles with different sizes around each local point, i.e. the deformation structure and the mechanical behavior of the assembly randomly generated by the particles with different sizes are related to microscopic structures of the granular assembly.

It is remarked that much more apparent influence of the rolling friction tangential force on the load-carrying capability is observed in the randomly generated granular assembly as shown in Figure 4(b) than that displayed in the assembly generated with homogeneous particles shown in Figure 4(a). It is also shown in Figure 4 that the rolling friction resistance increasing with the rolling stiffness coefficient $k_r$ enhances somewhat the load-carrying capability of the assembly.
6.2 A slope stability example

The second example concerned is a slope stability problem. The geometry of the slope is shown in Figure 12. The particles on the right boundary are free and fixed in the vertical and horizontal directions respectively. The particles on the bottom boundary are fixed in the vertical direction while their movements tangent to the bottom surface and their rolling movements are allowed with the same sliding, rolling friction coefficients and the same rolling friction moment coefficient, which are used for two typical particles in

**Figure 11.**
The effective strain distributions in the assembly randomly generated by particles of different radii with increasing vertical displacements prescribed to the top boundary of the assembly; (a) 1.2 cm; (b) 12 cm; (c) 30 cm; and (d) 43.2 cm.
contact, between the particles and the surface. The slope is loaded by a footing modeled as a rigid plate resting on its crest as shown in Figure 12. An increasing load is applied to the slope via the increasing vertical displacement prescribed to point A at the plate, so that the footing is also allowed to rotate around it. The material parameters used for calculation of the external forces applied to the particles contacted with the plate are the same as those used for two contacted particles. One of the main objectives of this example is to show the capability of the proposed discrete particle model in reproducing the different failure modes, i.e. the landslide, the compression cracking and the mud avalanching, which occur in the slope made of granular materials.

First, we consider the slope with geometrical sizes of $b = 50 \text{ cm}$, $H = 50 \text{ cm}$, $L = 25 \text{ cm}$, $a = 15 \text{ cm}$. The slope is modeled by an assemblage of 3,710 homogeneous particles with the radius of 5 mm collocated in a regular manner. The mechanical properties of the granular material used in this case are listed in Table II. Gravitational forces are taken into account. The rate of the increasing vertical displacement prescribed to point A is 69 cm/s and the incremental time step size $\Delta t \approx 1.441856 \times 10^{-5}$ s equals to

<table>
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</tr>
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<td>$2.0 \times 10^6$ (N/m)</td>
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<td>Stiffness coefficient of rolling force ($k_r$)</td>
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<td>Stiffness coefficient of rolling moment ($k_q$)</td>
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<tr>
<td>Sliding friction force coefficient ($\mu_s$)</td>
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<tr>
<td>Rolling friction force coefficient ($\mu_r$)</td>
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</tr>
<tr>
<td>Rolling friction moment coefficient ($\mu_q$)</td>
<td>$r\mu_q = 5.0 \times 10^{-5}$ m</td>
</tr>
</tbody>
</table>

Table II. The material parameters of the granular assembly used in the slope example with the failure mode of shear band
the critical time step size $\Delta t_{cr}$ calculated according to equation (38). Figure 13 shows the development of effective strains in the deformed slope with the increase of the prescribed vertical displacement of point $A$ at 3, 6 and 9 cm. The failure mode of the landslide, i.e. shear bands developed at first and then expanded to the surrounding zone can be observed. The latter phenomenon observed is different from the failure mode of shear bands occurring in the continuum model. It can be explained by the tendency of instability of the particles at the surrounding zone from their original positions due to the severe movements of the particles at the shear bands and the gravitational force effect.

Secondly, we consider the slope with the same geometry and modeling parameters of the granular assembly as in the first case. The material properties and the incremental time step size are also as same as those used in the first case except $c_r = c_s = c_g = 0$, i.e. viscous damping effects are neglected. Gravitational forces are also neglected. The slope is loaded by the enforced increasing vertical displacement prescribed to point $A$ with the rate 69 cm/s and the body force acceleration with prescribed increasing rate $226.4 \text{ m/s}^2$ until the time when the acceleration ...

Figure 13.
The failure mode: the landslide. Evolution of effective strains in the deformed slope with increasing prescribed vertical displacement of point $A$ equals to (a) 3 cm; (b) 6 cm; and (c) 9 cm.
10 m/s² (= 1 g) is reached and then the value of the acceleration is kept unchanged. Figure 14 shows the evolution of the deformation of the slope. It is remarked that the granular assembly is likely to deform as granular flow, i.e. the failure mode of the slope in this case is rather characterized by the mud avalanching while no intense strain localization concentrated into narrow shear bands is observed as shown in Figure 15.

Finally, the slope with geometrical sizes of $b = 60.5\text{ cm}$, $H = 35.5\text{ cm}$, $L = 30\text{ cm}$, $a = 20\text{ cm}$ is analyzed. The slope is modeled by 3,710 homogeneous particles with the radius of 5 mm collocated in a regular manner. The mechanical properties of the particles are determined by the effective stress and the Mohr-Coulomb failure criterion.

Figure 14. The failure mode: the mud avalanching evolution of the deformation of the slope with increasing prescribed vertical displacement of point A equals to (a) 3.3 cm; (b) 4.8 cm; (c) 7.8 cm; and (d) 10.8 cm.
The material parameters of the granular assembly used in the slope example with the cracking failure mode are listed in Table III. Gravitational forces are taken into account. The slope is loaded by the enforced increasing vertical displacement prescribed to point A with the rate 690 cm/s. The incremental time step size \( \Delta t = 1.441856 \times 10^{-6} \) s is used. The failure mode of the compression
cracking characterized by a number of macroscopic cracks running through the slope, including a vertical crack due to the compression, can be observed in Figure 16.

7. Concluding remarks
A systematic analysis of the kinematics of two moving circular grains with different radii in contact is carried out. Accordingly, the relative motion measurements, i.e. the relative rolling and sliding motion measurements, including the translational and angular velocities (displacements) are defined. Both the rolling and sliding frictional tangential forces, the rolling friction resistance moment, which are constitutively related to corresponding relative motion measurements defined, are formulated and integrated into the framework of dynamic model of DEM.

A discrete particle model for granular materials is proposed. In view of the fact that the rolling resistance mechanism described in currently available models in the literature is inadequate in both physical and numerical aspects, the proposed model is thus focused on an adequate description of the rolling resistance mechanism in the model. However the present work may be regarded as a further attempt to highlight the issue rather than to provide a comprehensive solution.

The definitions of nominal effective and volumetric strains are proposed on the formalism of the continuum theory to illustrate the movements of each particle relating to its surrounding particles in a particulate system.

The three types of failure modes, i.e. the landslide (shear bands), the compression cracking and the mud avalanching, in granular materials are reproduced to demonstrate the capability and the performance of the proposed discrete particle model and corresponding numerical method in the simulation of failure phenomena observed in granular materials.

Figure 16.
The failure mode: the compression cracking.
Evolution of effective strains in the deformed slope with increasing prescribed vertical displacement of point A equals to (a) 2.1 cm; and (b) 2.4 cm.
References


Feng, Y.T., Han, K. and Owen, D.R.J. (2002), “Some computational issues numerical simulation of particulate systems”, Fifth World Congress on Computational Mechanics, Vienna.


