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I.C.T.P., P.O. BOX 586, 34100 TRIESTE, ITALY, CABLE: CENTRATOM TRIESTE

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**AND SELF-ORGANIZING NANOSTRUCTURES**
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# "Capture Time in Quantum Well Laser Structure"

**K. KALNA**
**Institute of Electrical Engineering**
**Dubravska Cesta 9**
**84239 Bratislava**
**Slovak Republic**

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# Capture Time in Quantum Well Laser Structure

Karol KÁLNA

Institute of Electrical Engineering, Slovak Academy of Sciences
Dúbravská cesta 9, 842 39 Bratislava, Slovakia

1. Introduction to the capture time in a semiconductor laser structure
2. Investigated structure, ambipolar capture process
3. Electron capture time via the electron-polar optical phonon interaction
4. Theoretical interpretation of the measured carrier capture time by Blom et al (1993)
5. Electron capture time via the electron-electron interaction (with exchange effect)

T=8 K and $N_S = 10^{15}\mathrm{m}^{-2}$ to $10^{16}\mathrm{m}^{-2}$

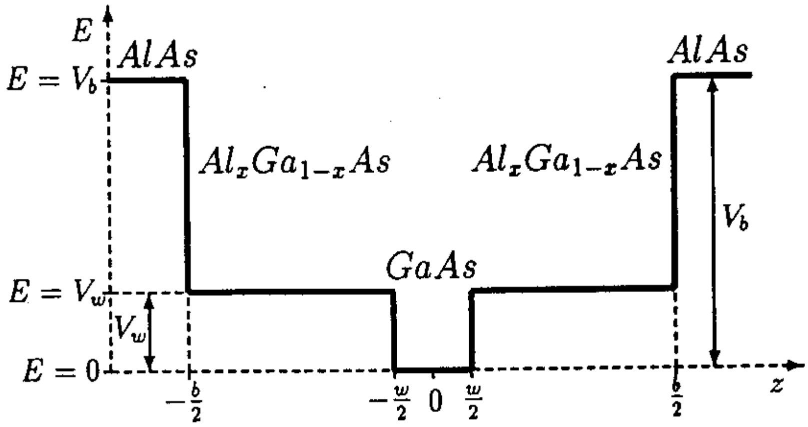



Figure 1: *Conduction-band edge diagram of the separate confinement heterostructure quantum well (SCHQW)*

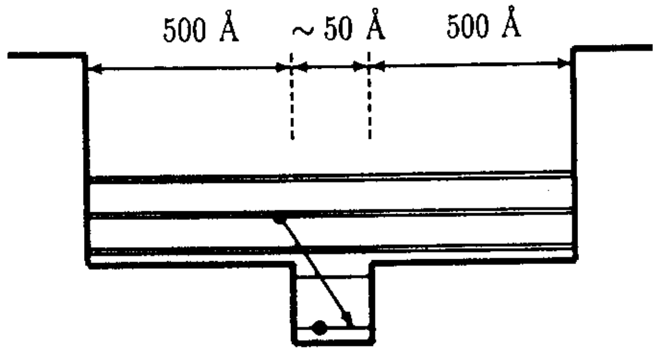



Figure 2: *Schematic view of a capture process to the quantum well in the SCHQW*

Electron scattering rates are based on the Fermi golden rule (Born approximation).

Transition probability from an initial state to a final state:

$$P\,(\text{ initial} \rightarrow \text{final }) = \frac{2\pi}{\hbar}\,|\langle \text{ initial} \mid H_{\text{interaction}} \mid \text{final}\rangle|^2\,\delta(E_{\text{final}} - E_{\text{initial}}),$$

$H_{\text{interaction}}$ - arbitrary interaction hamiltonian which is considered as a perturbation.

# Electron-Polar Optical Phonon Interaction

| | Initial State | | Final state | |
|---|---|---|---|---|
| | Subband | Wave vector | Subband | Wave vector |
| Electron | $i$ | $\mathbf{k}_1$ | $m$ | $\mathbf{k}_1'$ |

The electron-polar optical phonon scattering rate:

$$\lambda_{im}^{epop}(\mathbf{k}_1) = \frac{e^2 \omega_{LO} m_e}{8\pi\hbar^2} \left(\frac{1}{\kappa_\infty} - \frac{1}{\kappa}\right) \int_0^{2\pi} d\theta \, \frac{F_{iimm}^{ee}(q)}{q\, \epsilon^e(q)} \quad ,$$

$$q = \left[2k_1^2 + \frac{2m_e}{\hbar^2} E_S^e - 2k_1 \left(k_1^2 + \frac{2m_e}{\hbar^2} E_S^e\right)^{1/2} \cos\theta\right]^{1/2} \quad ,$$

$m_e$ - electron effective mass,

$E_\gamma^e$ $(\gamma = i, m)$ - electron subband energies,

$E_S^e = E_i^e - E_j^e - \hbar\omega_{LO}$,

$\omega_{LO}$ - frequency of longitudinal optical phonons,

$\kappa_\infty$ - high frequency permittivity,

$\kappa$ - static permittivity,

The static screening function is:

$$\epsilon^e(q) = 1 + (q_S^e/q) F_{1111}^{ee}(q)\; f_1^e(\mathbf{k}_1 = 0) \quad ,$$

$q_s^e = e^2 m_e / (2\pi\kappa\hbar^2)\; f_1^e(\mathbf{k}_2 = 0)$ - electron screening constant.

The form factors:

$$F_{ijmn}^{ee}(q) = \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2\; \chi_i^e(z_1)\, \chi_j^e(z_2)\, e^{-q|z_1 - z_2|}\, \chi_m^e(z_1)\, \chi_n^e(z_2) \quad ,$$

$\chi_\gamma^e$ - wave function of the electron in the subband $\gamma$ $(\gamma = i, j, m, n)$.

# Measurements of Carrier Capture Time

[Blom et al, PRB 47 (1993) 2072.]

Two basic experimental methods are used:

1. time-resolved luminescence spectroscopy (e.g. the upconversion technique)

   The capture time is determined from differences in the rise time of the QW luminescence after direct (below the barrier band gap) and indirect (above the barrier band gap) excitation with a subpicosecond laser pulse.

2. pump-probe spectroscopy

   The capture time is determined from the decay of barrier population in two-pulse correlation measurements, correlated luminescence of (i) the *AlGaAs* barrier and (ii) the GaAs buffer layer.

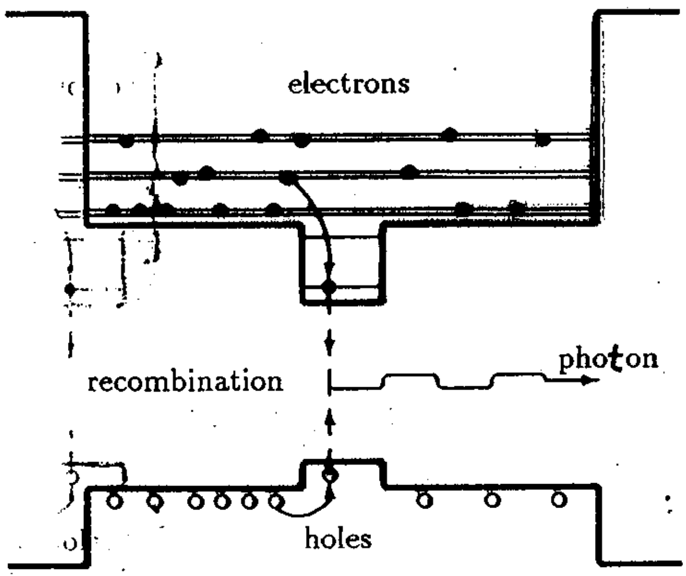



Figure 1: *Carrier capture processes from the conduction and valence bands for the optical experiments in the SCHQW*

# Electron Capture Time for the Optical Experiment by Blom

- electron temperature $T_i^e$ is assumed in the initial subband $i$

The electron distribution function $f_i^e(\mathbf{k}_1)$ is:

$$f_i^e(\mathbf{k}_1) \sim \exp \frac{\varepsilon(\mathbf{k}_1)}{k_B T_i^e},$$

$$k_B T_i^e = \frac{m_h^B}{m_e^B + m_h^B}(E_{\text{excess}} - E_i^e - E_i^h).$$

$E_i^e$ - initial electron energy in the subband $i$,
$E_i^h$ - initial hole energy in the subband $i$,
$m_e^B$ - electron effective mass in the barrier,
$m_h^B$ - hole effective mass in the barrier,
$E_{\text{excess}}$ - laser excess energy.

The optical matrix element in the absorption coefficient is proportional to

$$\begin{aligned} \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2\, \chi_i^{\alpha}(z_1)\, \chi_j^{\beta}(z_2) &\approx 1 \quad \text{if} \quad i = j; \qquad \alpha, \beta = e, h \\ &\approx 0 \quad \text{if} \quad i \neq j. \end{aligned}$$

which means that

$$n_i^e = n_j^e = n_i^h = n_j^h$$

in each subband, and the same number of electron and hole subbands is occupied.

The electron capture time is reciprocal of the electron capture rate:

$$\tau_{e-pop}^{-1} = \sum_{i,m,\mathbf{k}_1} \frac{f_i^e(\mathbf{k}_1)\, \lambda_{im}^{epop}(\mathbf{k}_1)}{f_i^e(\mathbf{k}_1)}.$$

# Hole Capture Time for the Optical Experiment by Blom

- classical model
- 3D-to-3D capture
- the 2D-to-2D capture model fails

Blom estimated the hole capture time $\tau_h = 12.5$ ps from the diffusion theory:

$$\tau_h = \frac{b^2}{\pi^2 D_h}, \qquad D_h = \frac{k_B T}{e}\mu_h,$$

where $\mu_h$ - measured in the n-AlGaAs bulk

$$\mu_h = 5.8 \cdot 10^3 \mathrm{cm^2 V^{-1} s^{-1}}$$

The diffusion concept is not appropriate because:

1. the hole mean free path $\ell_h$ is $\gg b/2$

$$\ell_h = v_h \tau_{\text{hole-impurity}} = v_h \frac{m_h^B \mu_h}{e} \approx 1000 \text{Å}, \qquad b/2 = 500\text{Å}$$

where $v_h$ is the typical hole velocity on the barrier.

Thus the hole mean free path in the samples of Blom et al is typically much larger than the barrier width.

2. the classical capture rate should in fact be

$$\frac{1}{\tau_h} \times \left( \begin{array}{c} \text{the probability that the hole emits op-} \\ \text{tical phonon when crossing the QW} \end{array} \right)$$

# Semiclassical Hole Capture Time for an Optical Experiment

- thermionic model with semiclassical transmissions coefficients

The semiclassical thermionic hole capture time is:

$$\tau_h^{-1} = b\left(\frac{\pi m_h'}{2k_BT}\right)^{1/2} T_{B\to W} \times \left[1 - \frac{1}{\pi}\int_{-\pi/2}^{\pi/2} d\phi \exp\left(-\frac{w}{\tau_{h-pop}^{(3D)}(E_h)\, v_h\, T_{W\to B}\cos\phi}\right)\right],$$

where the hole velocity $v_h$ on the barrier can be found from the laser excess energy as

$$v_h^W = \sqrt{2E_h/m_h}, \qquad E_h = \frac{1}{2}E_{\text{excess}}\frac{m_e}{m_e+m_h} + V_w^h,$$

$V_w^h$ - energy difference between the QW and barrier in the valence band,
$m_h$ - hole effective mass in the QW

The transmission coefficients are given by

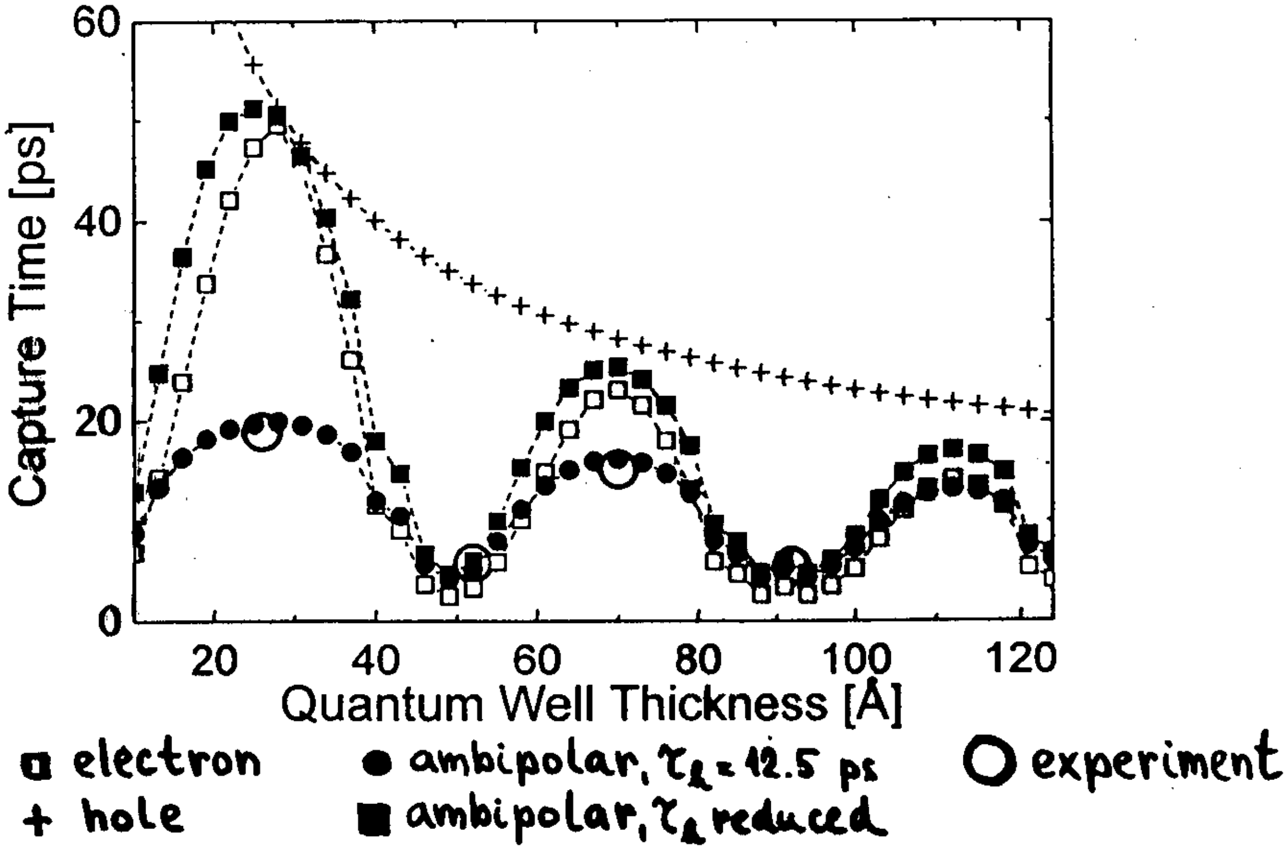
$$T_{B\to W} = T_{W\to B} = \frac{4\sqrt{m_h^B m_h (E_h - V_w^h)E_h}}{\left(\sqrt{m_h^B(E_h - V_w^h)} + \sqrt{m_h E_h}\right)^2},$$

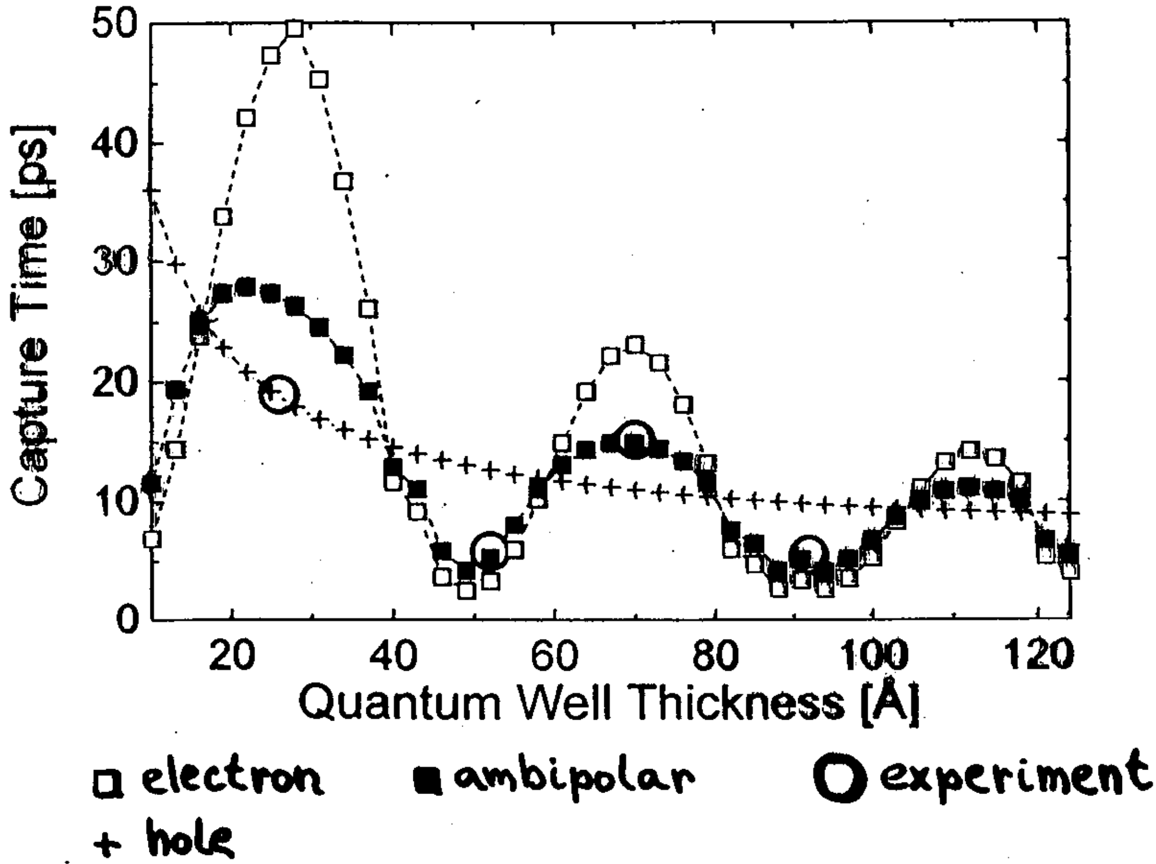
and the polar optical phonon emission rate of a hole by

$$\frac{1}{\tau_{h-pop}^{(3D)}(E_h)} = \sqrt{m_h/2}\,\frac{e^2\omega_{LO}}{4\pi\hbar}\left(\frac{1}{\kappa_\infty} - \frac{1}{\kappa}\right)\frac{1}{\sqrt{E_h}}\ln\left|\frac{\sqrt{E_h} + \sqrt{E_h - \hbar\omega_{LO}}}{\sqrt{E_h} - \sqrt{E_h - \hbar\omega_{LO}}}\right|.$$

The ambipolar capture time reads:

$$\frac{1}{\tau_a} = \frac{1}{2}\left(\frac{1}{\tau_e} + \frac{1}{\tau_h}\right).$$


Capture Time [ps]
Quantum Well Thickness [Å]
electron
hole
ambipolar, τ = 12.5 ps
ambipolar, τ reduced
experiment


Capture Time [ps]
Quantum Well Thickness [Å]
electron
hole
ambipolar
experiment

# Electron-Electron Interaction

| | Initial State | | Final state | |
|---|---|---|---|---|
| | Subband | Wave vector | Subband | Wave vector |
| First Electron | $i$ | $\mathbf{k}_1$ | $m$ | $\mathbf{k}_1'$ |
| Second Electron | $j$ | $\mathbf{k}_2$ | $n$ | $\mathbf{k}_2'$ |

The electron capture time for the laser regime is reciprocal of the electron capture rate

$$\tau_{e-\eta}^{-1} = \frac{\sum_{i,m,\mathbf{k}_1} f_i^e(\mathbf{k}_1)\ \lambda_{im}^{e\eta}(\mathbf{k}_1)}{\sum_{i,\mathbf{k}_1} f_i^e(\mathbf{k}_1)} \quad , \qquad \eta = pop, e,$$

$i -$ subband $>$ $AlGaAs$ barrier,

$m -$ subband $<$ $AlGaAs$ barrier,

$f_i^e(\mathbf{k}_1)$ - electron distribution in the subband $i$.

The electron-electron scattering rate of an electron with wave vector $\mathbf{k}_1$ from the subband $i$ to the subband $m$:

$$\lambda_{im}^{ee}(\mathbf{k}_1) = \frac{1}{N_S A} \sum_{j,n,\mathbf{k}_2} f_j^e(\mathbf{k}_2)\ \lambda_{ijmn}^{ee}(g),$$

$g = |\mathbf{k}_1 - \mathbf{k}_2|$ - relative vector,

$i -$ subband $>$ $AlGaAs$ barrier,

$j -$ , $m -$ and $n -$ subbands $<$ $AlGaAs$ barrier,

$A$ - normalization area,

$f_j^e(\mathbf{k}_2)$ - Fermi-Dirac distribution function of the electron in the subband $j$ at temperature 8 K and for electron density $N_S$.

1) The **electron-electron** pair scattering rate:

$$\lambda_{ijmn}^{ee}(g) = \frac{N_S m_e e^4}{16\pi\hbar^3\kappa^2} \int_0^{2\pi} d\theta\ \frac{|F_{ijmn}^{ee}(q)|^2}{q^2\ \epsilon^e(q)^2} \quad ,$$

$$q = \frac{1}{2}\left[2g^2 + \frac{4m_e}{\hbar^2}E_S^e - 2g\left(g^2 + \frac{4m_e}{\hbar^2}E_S^e\right)^{1/2} \cos\theta\right]^{1/2},$$

and

$$E_S^e = E_i^e + E_j^e - E_m^e - E_n^e,$$

$$\epsilon^e(q) = 1 + (q_S^e/q)F_{1111}^{ee}(q)\ f_1^e(\mathbf{k}_1 = 0).$$

The **electron-electron** multisubband pair scattering rate:

$$\lambda_{ijmn}^{ee}(g) = \frac{N_S m_e e^4}{16\pi\hbar^3\kappa^2}\int_0^{2\pi} d\theta \left[F_{ijmn}^{ee}(q) - \frac{q_s^e}{q\epsilon^e(q)}F_{i1m1}^{ee}(q)F_{1j1n}^{ee}(q)\right]^2 q^{-2}.$$

The assumption:

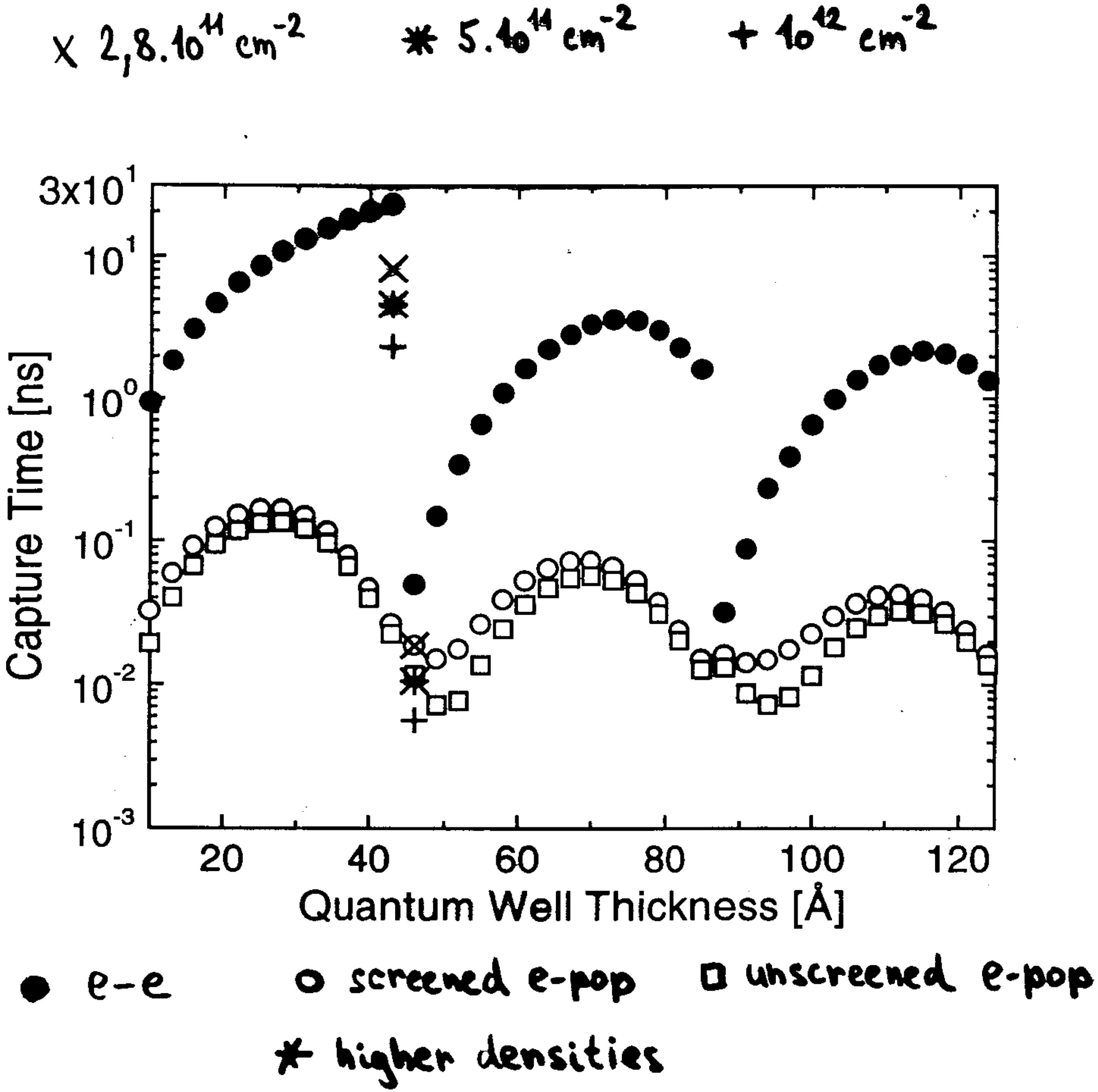
$$\frac{F_{ijmn}^{ee}(q)F_{1111}^{ee}(q)}{F_{i1m1}^{ee}(q)F_{1j1n}^{ee}(q)} \approx 1.$$

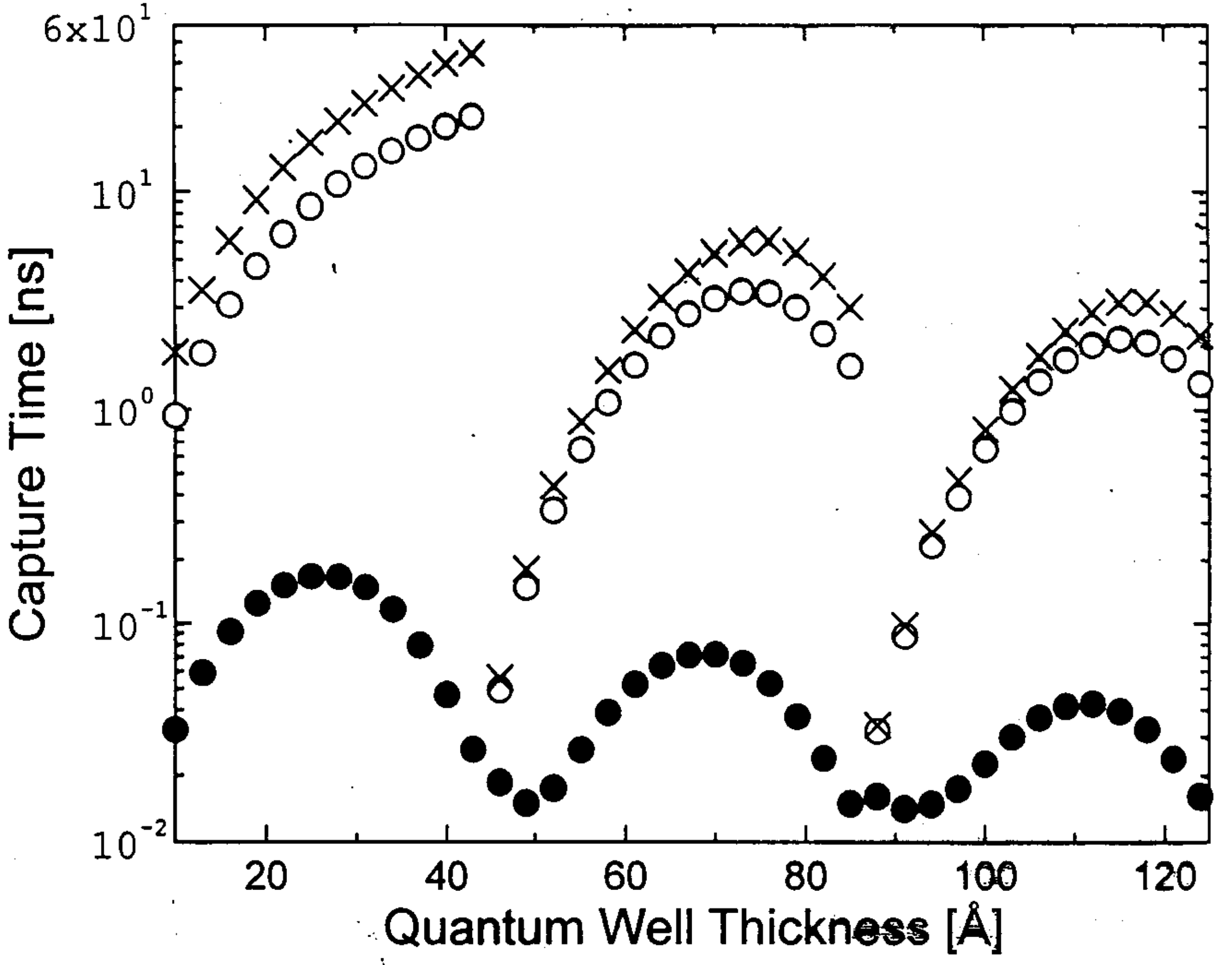
Incorporation of the exchange into the e-e scattering rate:

$$\frac{|F_{ijmn}^{ee}(q)|^2}{q^2\epsilon^e(q)^2} \mapsto \frac{1}{2}\left[\frac{|F_{ijmn}^{ee}(q)|^2}{q^2\epsilon^e(q)^2} + \frac{|F_{ijnm}^{ee}(q')|^2}{q'^2\epsilon^e(q')^2} - \frac{F_{ijmn}^{ee}(q)F_{ijnm}^{ee}(q')}{q\epsilon^e(q)q'\epsilon^e(q')}\right],$$

where

$$q' = \frac{1}{2}\left[2g^2 + \frac{4m_e}{\hbar^2}E_S^e + 2g\left(g^2 + \frac{4m_e}{\hbar^2}E_S^e\right)^{1/2}\cos\theta\right]^{1/2}.$$


X 2,8.10^14 cm^-2
* 5.10^14 cm^-2
+ 10^12 cm^-2
3x10^1
10^1
10^0
10^-1
10^-2
10^-3
Capture Time [ns]
20
40
60
80
100
120
Quantum Well Thickness [Å]
● e-e
o screened e-pop
□ unscreened e-pop
* higher densities




● screened e-pop   ○ e-e without exchange
× e-e with exchange

## Conclusions

- The electron and hole capture times in the SCHQW have been calculated.

- The electron capture time via the electron-electron interaction plays a role only in the resonances, and at sheet carrier densities in the QW close to $10^{12}\text{cm}^{-2}$.

- The hole capture time has been calculated semiclassically using the thermionic model.

- Qualitatively, the measured capture times by Blom seem to be well understood, but quantitative understanding is still missing.

One should consider:

1. the carrier capture via confined phonons
2. the hole capture via non-polar optical interaction
3. the quantum treatment of the hole capture beyond the assumption of coherent 2D barrier states