



The University of Sheffield

Department of Mechanical Engineering

Dynamics Research Group

# Robust Reliability of Neural Networks Using Information-Gap Models

University of Bristol

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Prof Keith Worden

Dr S.G.Pierce, Dr G. Manson

*Department of Mechanical Engineering,  
University of Sheffield  
Mappin Street, Sheffield S1 3JD, UK*

*Aknowledgements:*

*EPSRC*

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*NETLAB and INTLAB for MATLAB™*



# Introduction

- ❑ Background: classification of acoustic and ultrasonic signals used for structural damage detection
- ❑ Previous work (at Sheffield) has focussed on
  - ❑ Neural networks
  - ❑ Outlier analysis
  - ❑ Sammon mapping
  - ❑ Wavelet analysis
- ❑ All successful at categorising damage, but...
- ❑ Potential problems with signal variability & noise (environmental and instrumentation)
- ❑ Current work investigates robustness of neural network for a classification problem

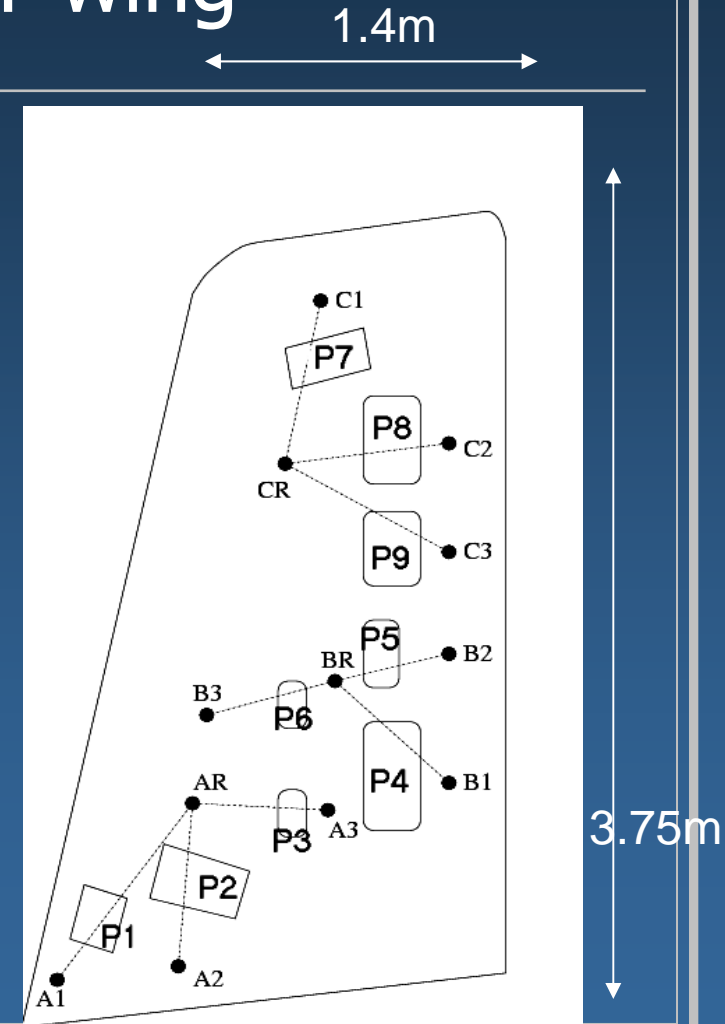


# Background

- ❑ Application area, interpretation and classification of acoustic and ultrasonic data used for health monitoring
- ❑ For example, complex signals arising from
  - ❑ Vibration / modal analysis
  - ❑ Ultrasonic guided waves,
    - ❑ Multiple modes
    - ❑ Mode conversion
    - ❑ Environmental effects
    - ❑ True damage / defects
- ❑ Resistant to simple time domain analysis

# Example data GNAT wing

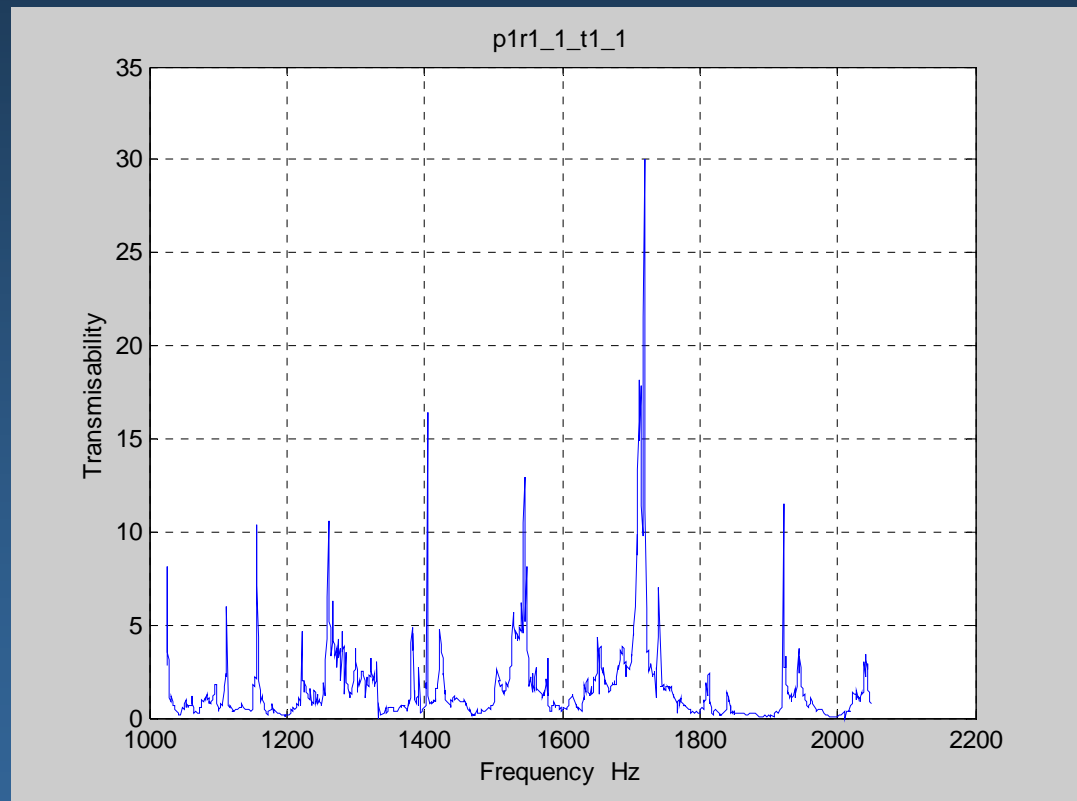
- 12 accelerometers measuring forced vibration of wing
- 9 removable panels to simulate damage
- Measure transmissibilities between transducer pairs





# Raw transmissibility data

- 1024 spectral lines
- 1024-2048 Hz



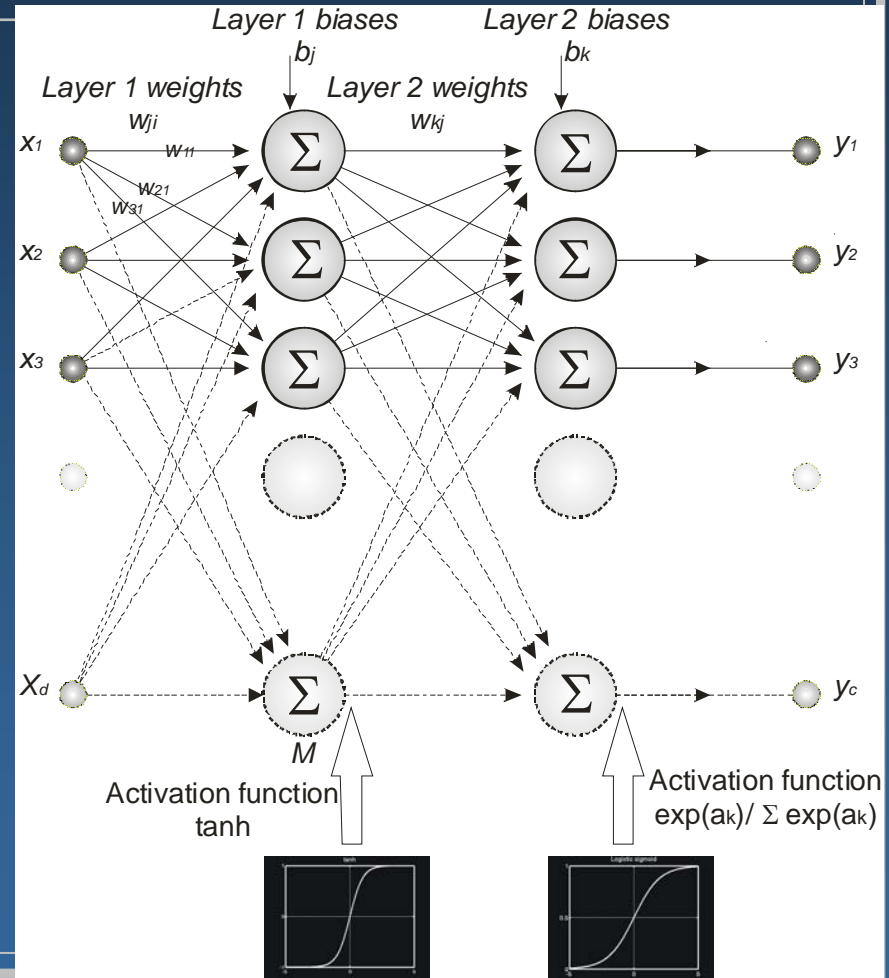


# Outlier analysis

- ❑ Manual selection of features from the transmissibility spectra
- ❑ Reduced data set to a “best feature” corresponding to removal of a particular panel
- ❑ For each feature there were 1800 test measurements and 700 normal condition measurements
- ❑ Outlier analysis performed to generate matrix of novelty values
- ❑ Data divided into training, validation and test sets

# MLP Network (implemented using NETLAB)

- 9 inputs corresponding to 9 selected features (doesn't have to be 9)
- 9 outputs corresponding to the 9 different damage conditions (panel removals)
- 1 hidden layer with variable number of nodes
- Softmax output layer
- Weight Decay regularisation





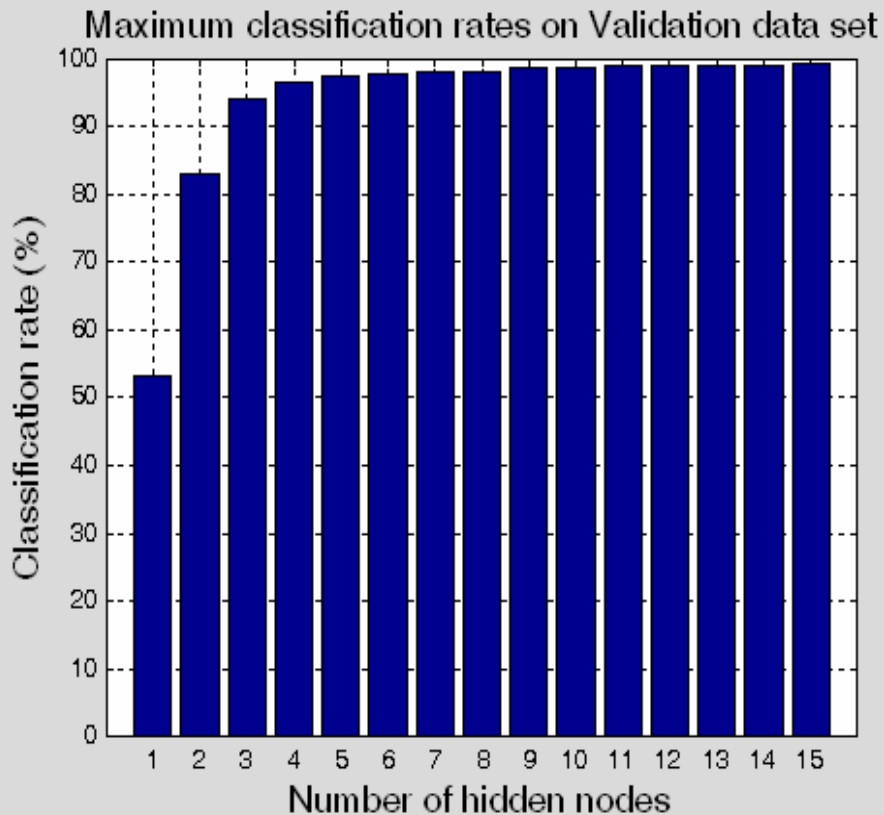
# Conventional Network Training and Performance

- ❑ Multiple network structures (100 independent networks) trained on the training data with hidden nodes = 1 to 15
- ❑ Validation data used to select best performing network
- ❑ Test data used to assess network performance





# Network performance (validation data set)



- Performance judged on validation data set
- $n\_hidd=4$  gave (highest) 97.3 % correct classification rate on validation set
- Sparse data therefore overtraining danger



# Network Performance (4 hidden nodes)

	TRAINING		VALIDATION		TEST	
	min	max	min	max	min	max
Classification Rate (%)	96.1	100	90.7	97.3	87.7	93.6
Network Number	25	4	21	85	48	71



# Network performance (test data set)

Classification rate=92.9293 %

61	1	0	0	1	1	2	0	0
2	60	2	0	1	1	0	0	0
1	0	58	3	0	2	0	2	0
0	0	0	63	1	1	0	1	0
1	2	0	2	61	0	0	0	0
0	0	4	1	0	61	0	0	0
0	0	0	0	0	0	66	0	0
0	0	0	1	0	0	3	61	1
0	0	0	0	0	0	2	3	61

Test data: nhidd=4; ncyc=85

Performance of test data through  
best network (#85) selected from  
validation data



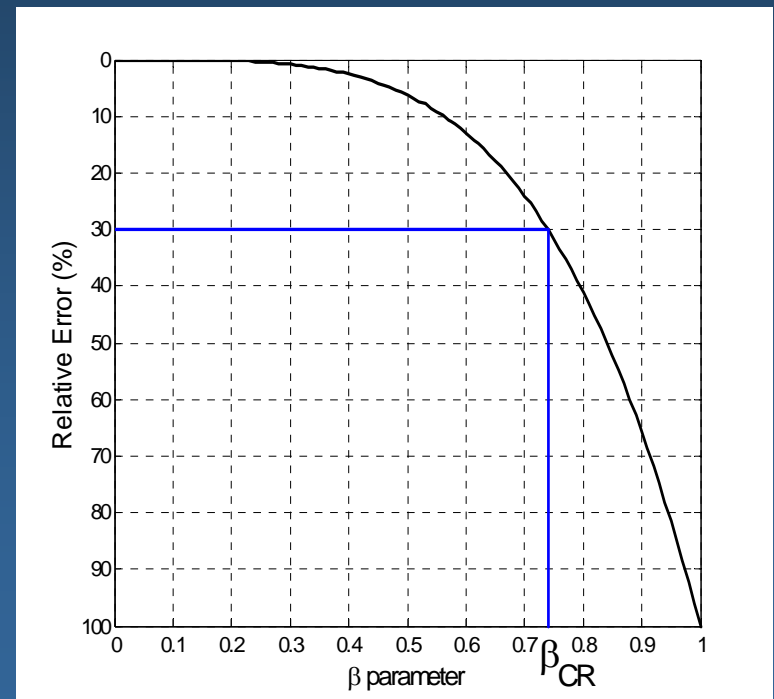
# Evaluation of network "robustness"

- We are interested in how a particular network will perform when the input data becomes noisy
- However real test data noise may be statistically very different to the noise encountered during the training phase; we want to have confidence in network classification performance under these conditions
- For example in a safety critical application we need a quantitative evaluation of the likelihood of misclassification
  - Probabilistic techniques can provide confidence bounds, but these are not guarantees, only probabilities
  - Info-gap analysis provides a definite bound to a given level of input uncertainty, useful if we specify that a particular misclassification is unacceptable in all cases



# Quantifying network reliability

- With each network, we associate an input set  $I(\beta)$  composed of all possible inputs to the network. The size of uncertainty is described by the  $\beta$  parameter.
- Given the set of inputs, we compute the response set  $R(\beta)$  of all network outputs
- The network reliability is related to how large  $\beta$  can be before a point in the failure set is reached
- Critical value  $\beta_{CR}$  defines attaining the failure set
- Large  $\beta_{CR}$  value is desirable, network is robust





# Interval number sets

- Use interval numbers to define an input set to the network under test
  - $x$  is a point on the number line bounded by  $b, a$
  - The interval number  $[a, b]$  is defined

$$[a, b] = \{x \mid a \leq x \leq b\}$$



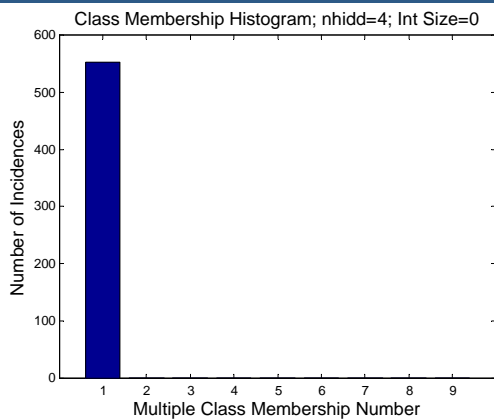
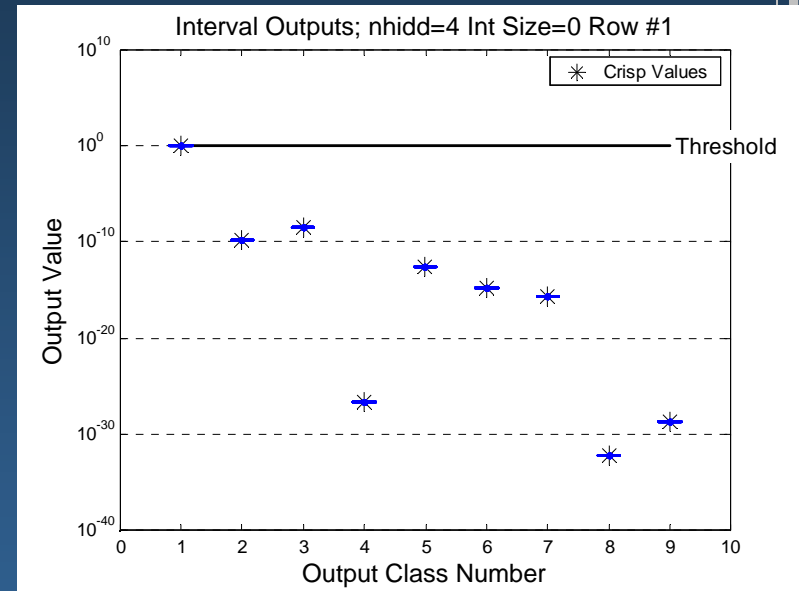
- So the network inputs are interval numbers

$$[x_{ia}, x_{ib}] = [(x_i - \beta), (x_i + \beta)]$$



# Worst case error and opportunity for classification problems

- Interval size = 0 (degenerate with CRISP outputs)
- Output prediction
  - Single class membership

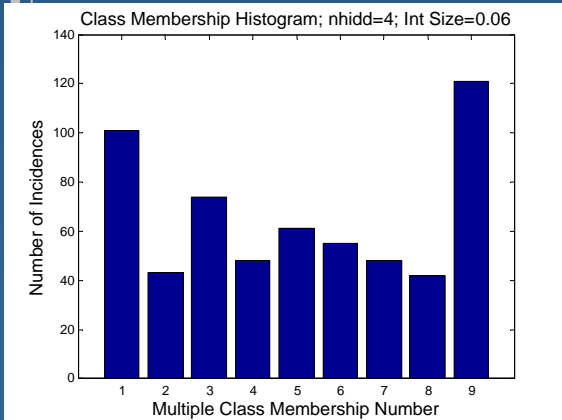
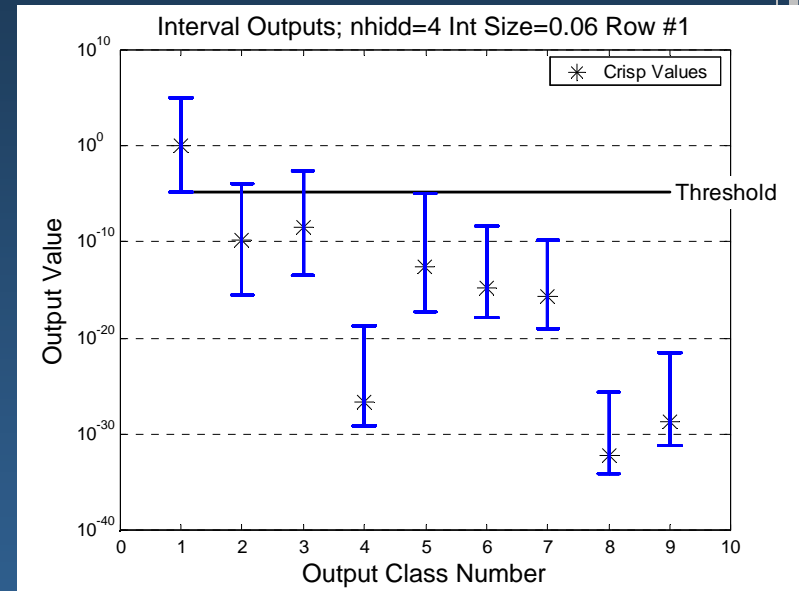


Target = class 1



# Worst case error and opportunity for classification problems

- Interval size = 0.06
- Output prediction
  - 3 class membership
  - Threshold set as the minimum of the interval with the greatest maximum



Target = class 1





# Definitions for interval classification problems

- As output interval size increases
  - More likely for output set to contain target
    - Best case classification rate increases
  - However, multiple class membership leads to increasing uncertainty in classification
    - Worst case classification rate decreases
    - Define **WORST CASE** as percentage number of total hits minus number of hits with multiple class membership
    - Could account for class membership number (not done here), i.e. it's miss-classification probability increases with class membership number
  - **OPPORTUNITY** measures the improvement (headroom) over the crisp classification rate

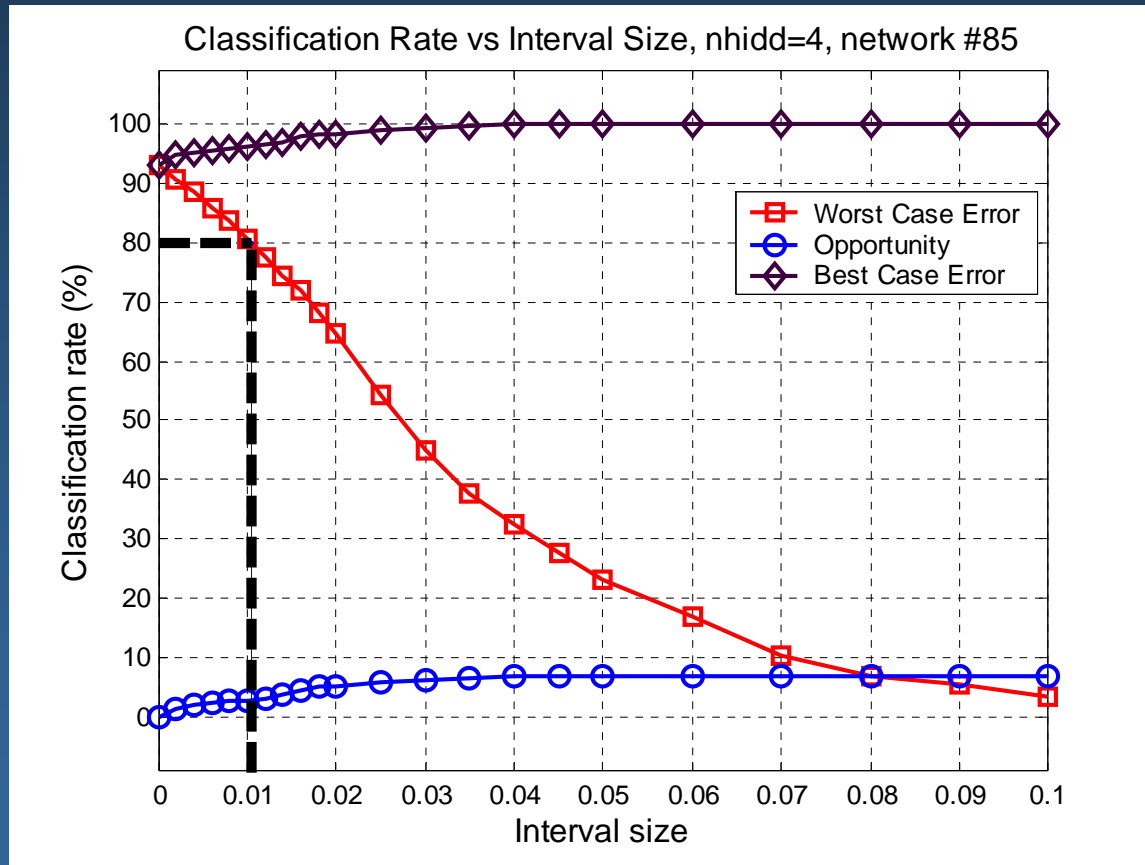


## Definitions for interval classification problems

- ❑ **BEST CASE** = The percentage of total correct classifications or hits, irrespective of class membership function
- ❑ **WORST CASE** = The percentage number of total hits minus number of hits with multiple class membership
- ❑ **OPPORTUNITY** = Best case – crisp classification rate



# Classification rate against interval size (robustness of a single network)





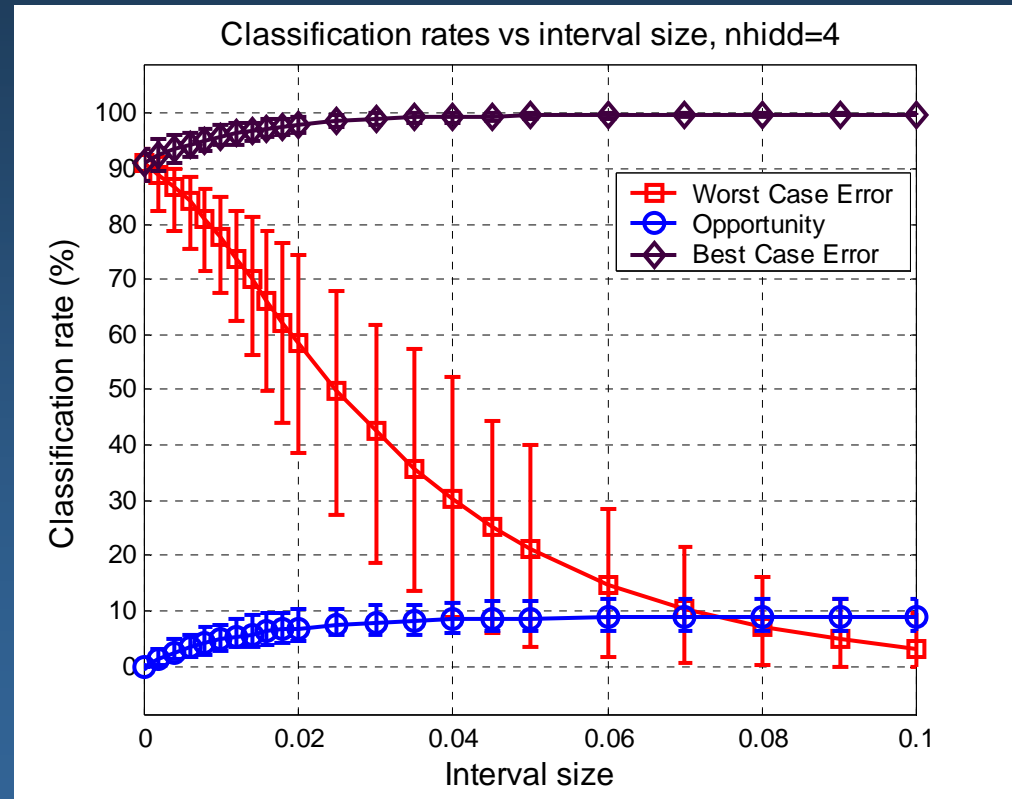
## Interval based network selection

- Previous example used interval propagation to investigate robustness of a particular network
- HOWEVER...
  - Can also use interval propagation to select the most robust network from many possibilities



# Interval Output Variability Across Multiple Networks

- Variation across 100 networks showing mean (centre markers) and range





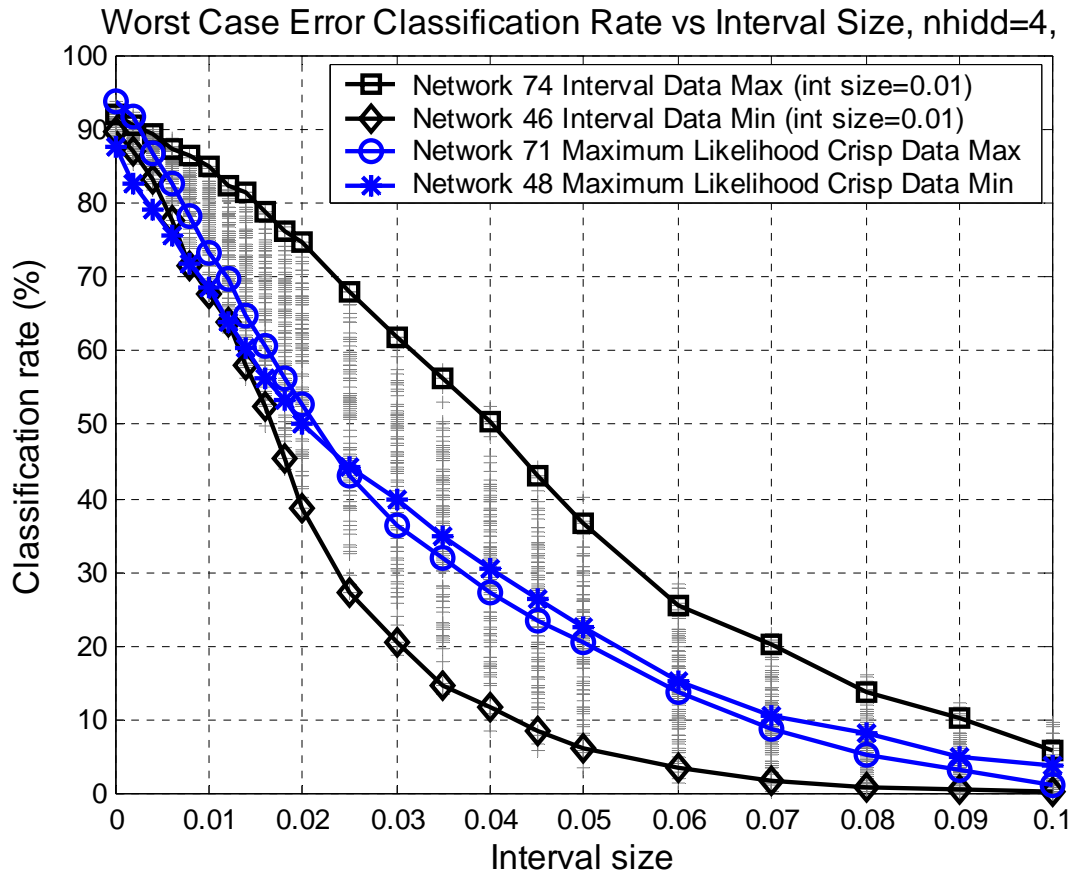
# Interval Output Variability Across Multiple Networks

- Best network performance depends on interval size

Interval Size	Minimum Worst Case Error		Maximum Worst Case Error	
	Value	Network Number	Value	Network Number
0	87.71	48	93.60	71
0.002	82.49	48	92.26	10
0.004	78.96	48	89.90	93
0.006	75.59	48	88.72	93
0.008	71.38	46	86.36	57
<b>0.010</b>	<b>67.68</b>	<b>46</b>	<b>84.85</b>	<b>74</b>
0.012	62.46	80	82.32	57
0.014	56.40	31	81.31	74
0.016	50.00	31	78.79	74
0.018	43.94	31	76.77	93
<b>0.020</b>	<b>38.72</b>	<b>46</b>	<b>74.58</b>	<b>74</b>
0.025	27.44	46	68.01	74
0.030	18.86	26	61.78	74
0.035	13.47	26	57.41	93
0.040	8.75	26	52.53	93
0.045	5.89	26	44.44	93
0.050	3.54	26	40.07	93
0.060	1.52	26	28.62	93
0.070	0.51	26	21.72	93
0.080	0.34	26	16.33	49
0.090	0.00	100	12.29	49
0.100	0.00	66	9.76	21



# Interval Improvement Over Best Crisp Network





# Conclusions

- ❑ Standard MLP used for classification problem
- ❑ Low frequency (1-2 kHz) vibration data from GNAT wing
- ❑ Non-probabilistic approach provides a conservative (Robust) estimate of worst case error due to input perturbations
  - ❑ Interval based Information-Gap technique provides...
    - ❑ Single network robustness quantification
    - ❑ Multiple network selection procedure
      - ❑ Significant improvement over crisp network training