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Analysis of energy harvesters for highway bridges

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Abstract

This article investigates the possibility of piezoelectric energy harvesters as energy scavenging devices in highway bridges. The structural vibration due to the motion of a load (vehicle) on the bridge is considered as the source of energy generation for the harvester. The energy generated in this way can be useful for wireless sensor networks for structural health monitoring of bridges by reducing or even eliminating the need for battery replacement/recharging. A highway bridge model with a moving point load is investigated and a linear single-degree-of-freedom model is used for the piezoelectric energy harvester. Two types of harvesters, namely, the harvesting circuit with and without an inductor, have been considered and the energy generated for a single vehicle has been estimated. These results may be used, together with traffic statistics, to obtain the variation of average power and thus, for a given application, help to design the energy management system.

Keyword

energy harvesting, highway bridge, moving load, piezoelectric energy harvesters, structural health monitoring

Introduction

In order to keep structures safe and more durable, novel sensing and control technologies and analytical methods are pursued by the engineering community. This helps to rapidly identify the onset of structural damage in an instrumented structural system or to control a structure (Ali and Ramaswamy, 2009). Structural health monitoring (SHM) systems can be found in a number of structures including aircraft, ships, and civil structures. To address the limitations of the current wired sensing technologies, the engineering and research communities are exploring new technologies such as wireless sensors and networks that can be placed at remote locations, providing a global view of the structural performance (Lynch and Loh, 2006; En et al., 2010; Park et al., 2010).

Wireless structural monitoring systems are inexpensive to install because wiring is no longer required between the sensors and the data acquisition system. These low costs promise the use of more sensors and more densely located sensors as compared to traditional monitoring systems. Hundreds of wireless sensors can be installed in a single structure (Park et al., 2010), providing better insight to structural damage by monitoring the behavior of critical structural components, thereby implementing local damage detection. The structural vibration signature can be used to identify the location and extent of damage.

Practical examples of civil infrastructure that include wireless sensing systems are few but the number is increasing rapidly. Rice et al. (2010) reported health monitoring using the 'Imote2' smart sensor platform on the Stawamus Chief Pedestrian Bridge (located in Vancouver in British Columbia, Canada). Practical aspects of SHM of a building with acoustic and vibration wireless sensors are detailed by En et al. (2010). Park et al. (2010) reported SHM systems using wireless smart sensor networks for a cable-stayed bridge (484 m long 2nd Jindo Bridge in Korea). Park et al. (2010) highlighted the need for energy harvesting devices to power the wireless sensor nodes or to switch to completely self-powered sensor nodes. Out of a total of 70 sensor nodes installed to assess the integrity of a bridge, five sensor nodes on the cables are powered by solar energy harvesting, whereas wind-based energy harvesters are used for some of the sensors on the underside of the bridge.

Providing power to the wireless sensors or sensor networks is an engineering challenge. Wired power is either prohibitively expensive (e.g. for civil structure) or too risky (e.g. for implanted medical sensors), while batteries require recharging/replacement and might be

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impractical in many scenarios. For practical infrastructure monitoring, it is desirable that the sensors last the service lifetime of the structure; typically, a structure's life span is several decades, but it can be as long as 100 years. In the cases of remote structures and embedded sensors, it is impractical and costly to change batteries periodically through the lifetime of the structure.

Therefore, engineers have sought methods to generate power from ambient sources, such as solar energy, temperature variation, or structural vibration, to recharge the batteries or to design self-powered sensors (energy harvesters) that scavenge energy available in the environment. Reviews on energy harvesting from mechanical and biological systems are given by Anton and Sodano (2007), Priya (2007), and Sodano et al. (2004). Applications include wireless sensor systems that are desirable in biological implants, robotic devices, and SHM, where remote operations are required.

Most energy harvesting techniques investigated and implemented for SHM are based on solar energy, thermal gradients, and/or vibration energy. The most successful energy harvesters to date are solar powered. However, SHM sensors could be embedded within concrete structures and therefore, it is not always feasible to use solar power. Heat or thermal gradients produce energy at very low levels and are unlikely to power sensors for infrastructure monitoring (Roundy, 2003). Currently, several research groups are developing energy harvesters based on ambient vibration energy (Roundy, 2003; Elvin et al., 2006; Sazonov et al., 2009). However, vibration energy harvesting from bridge vehicle interaction is still in its infancy and is the main theme of this article.

Various concepts to harvest energy from ambient vibration of the host system have been proposed (Sodano et al., 2004; Lefeuvre et al., 2005, 2006; Beeby et al., 2006; Anton and Sodano, 2007; Priya 2007; Ali et al., 2011). The two main vibration-based energy harvesting technologies are electromagnetic and piezoelectric. The electromagnetic harvester generates power from the relative motion of a coil due to host vibration in a magnetic field (Williams and Yates 1996; Amirtharajah and Chandrakasan 1998; Kulkarni et al., 2008). Piezoelectric energy harvesters generate power from the strain in piezoelectric materials in response to external mechanical vibrations (Tanner and Inman, 2002; Sodano et al., 2005; Adhikari et al., 2009; Ali et al., 2010). The advantages of piezoelectric devices include small size, fewer moving parts, and simpler design. The power generated by these harvesters are such that it can power a magneto-rheological damper (Tanner and Inman, 2002).

The aim of this article is to investigate energy harvesting technology for bridges with moving loads, which are able to provide a continuous source of energy scavenging. A single degree of freedom (SDOF) piezoelectric energy harvester is considered for the analysis. Although the parameters for the harvester considered represent a piezoelectric harvester, the same system equations and analysis may be used for electromagnetic harvesters (Halvorsen, 2008). The article is organized as follows: Section 2 details the analytical derivation for a beam with a moving point load. Fourier transforms are used to obtain an explicit expression of the response of the beam at any location. Section 3 outlines the details of piezoelectric energy harvesters with and without an inductor. Explicit expressions for the voltage and power obtained for the harvesters with and without an inductor are given. Section 4 discusses the optimal harvester location and parameters. Finally, the numerical results and conclusions are drawn in Sections 5 and 6, respectively.

The moving load problem

To model the dynamics of a bridge with a moving vehicle, we consider a beam with a moving point load (Fryba, 1999), as shown in Figure 1. This model has been validated by Stancioiu et al. (2011) and is the simplest model that includes the bridge dynamics. Other factors may be included, but that would complicate the system dynamics. For example, the vehicle suspension dynamics could be included, which would lead to a larger range of excitation frequencies at the harvester location (Li et al., 2008). However, the dynamics would vary considerably between vehicles, which means that the harvester could not be tuned to a single resonance frequency. In contrast, the bridge resonance frequencies will be relatively constant, apart from relatively small changes due to temperature, humidity, and other environmental factors. Furthermore, for long-span bridges, the vehicle natural frequencies are likely to be much higher than the lower bridge natural frequencies. Other

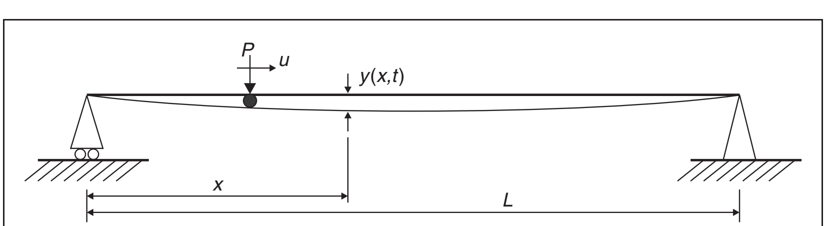

Figure 1. Schematic diagram of a beam with a moving point load.

phenomena that produce vibration could also be used to harvest energy, such as the roughness of the road surface or wind excitation; such excitation typically has a relatively high frequency and a relatively low amplitude, and is not considered further in this article.

The dynamics of a beam with a point load, P, moving at a velocity, u, is given by:

$$m\frac{\partial^2}{\partial t^2}y(x,t) + c\frac{\partial}{\partial t}y(x,t) + EI\frac{\partial^4}{\partial x^4}y(x,t) = \delta(x - ut)P \tag{1}$$

where $y(x, t)$ is the displacement of the beam at position x and at time t, m the mass per unit length, and L the length of the beam. The damping and the flexural rigidity of the beam are given by c and EI, respectively. Note that x is only defined for positions on the bridge, and hence $0 \le x \le L$. Thus, the Dirac delta function on the right-hand side of Equation (1) ensures that the force P is only applied to the bridge when the moving load is on the bridge. Equation (1) has to be supplemented with appropriate initial and boundary conditions and these will be considered later.

Since Equation (1) is a linear partial differential equation (PDE), a solution is assumed of the form:

$$y(x,t) = \sum_{j=1}^{N} \phi_j(x) q_j(t), \tag{2}$$

where $\phi_j(x)$ is the jth mode shape function of the beam and $q_j(t)$ the jth modal displacement. Note that the exact solution for the beam has an infinite number of modes, which would require an infinite series for the general solution; Equation (2) represents this series truncated to N terms. The mode shapes, $\phi_j(x)$, are independent of time, t, and satisfy the boundary conditions, whereas the modal displacements $q_j(t)$ are independent of the spatial variable x and satisfy the initial conditions of the PDE. The mode shapes are assumed to be mass normalized so that:

$$\int_0^L m\phi_j^2(x)\mathrm{d}x = 1. \tag{3}$$

Substituting Equation (2) into Equation (1), premultiplying by each mode shape in turn, integrating along the spatial domain from 0 to L, and imposing the orthogonal property of the mode shapes gives a set of N independent ordinary differential equations (ODEs). The jth ODE, corresponding to the jth modal displacement $q_j(t)$, is:

$$\ddot{q}_j + 2\zeta_j\omega_j\dot{q}_j + \omega_j^2 q_j = P\phi_j(ut) \tag{4}$$

where ω_j is the jth natural frequency of the system given by $\omega_j^2 = \int_0^L EI(\phi_j''(x))^2\mathrm{d}x$, and the prime denotes differentiation with respect to x. The analytical expressions for ω_j depend on the boundary conditions of the beam. The damping of the j th mode is given by ζ_j (assuming classical damping) and from Equation (1) is given by $2\zeta_j\omega_j = c/m$; this approach also allows for a variety of damping models to be approximated using the equivalent modal damping ratios. In Equation (4), the mode shape $\phi_j(x)$ is defined to be zero for $x > L$ so that the force is only applied while the vehicle is on the bridge.

Note that Equation (4) is valid for any beam (uniform or non-uniform) and with any boundary condition, provided the $\phi_j(x)$ satisfies these boundary conditions. An equivalent set of ODEs may be obtained for any general finite element model of the bridge, provided $\phi_j(ut)$ is interpreted as the vertical displacement for the j th mode at the location of the application of the point load.

Transformation to the frequency domain

Domain transformations are widely used to analyze moving load problems on an elastic beam; for example, Fryba (1999) and Lv et al. (2010) used the Laplace transform, whereas Sun (2001) used the Fourier transformation. Taking the Fourier transform of Equation (4), we obtain:

$$\left(-\omega^2 + 2\zeta_j\omega_j\mathrm{i}\omega + \omega_j^2\right)Q_j(\omega) = P\int_0^{L/u} e^{-\mathrm{i}\omega t}\phi_j(ut)\mathrm{d}t \tag{5}$$

where $Q_j(\omega)$ is the Fourier transform of $q_j(t)$ and $\mathrm{i} = \sqrt{-1}$. Note that the upper limit of the integral defining the modal force on the right-hand side of Equation (5) is not infinity but the time when the vehicle leaves the bridge. An expression for this modal force may be obtained explicitly in many cases or alternatively for a given mode shape may be obtained by numerical integration.

The displacement of the beam in the frequency domain at location x is then given by:

$$Y(x,\omega) = \sum_{j=1}^{N} \phi_j(x) Q_j(\omega). \tag{6}$$

Solution for a simply supported beam

For simply supported uniform beams, the mass normalized mode shapes are given by sine functions (Meirovitch, 1986; Chopra, 2001); therefore, $\phi_j(x) = \sqrt{\frac{2}{mL}}\sin(j\pi x/L)$. Thus, we have $\omega_j = \frac{j^2\pi^2}{L^2}\sqrt{\frac{EI}{m}}$ and $\zeta_j = \frac{c}{2m\omega_j}$. For a simply supported beam, the critical velocity of the vehicle at which beam resonance is expected is $u_c = \frac{\pi}{L}\sqrt{\frac{EI}{m}} = \frac{L\omega_1}{\pi}$.

The Fourier transform of the j th modal response, $Q_j(\omega)$, is given by Equation (5) as:

$$Q_j(\omega) = P\sqrt{\frac{2}{mL}}\frac{(j\pi u/L)\left(1 - (-1)^j e^{-\mathrm{i}\omega L/u}\right)}{(-\omega^2 + (j\pi u/L)^2)(-\omega^2 + 2\mathrm{i}\omega\zeta_j\omega_j + \omega_j^2)} \tag{7}$$

The $-\omega^2 + 2\mathrm{i}\omega\zeta_j\omega_j + \omega_j^2 = 0$ term in the denominator ensures that the modal response will usually have peaks at the bridge's natural frequencies. Although the denominator is zero at frequencies that depend on the vehicle speed given by $\omega = \pm j\pi u/L = \pm u\omega_j/u_c$, the numerator is also zero at these frequencies and the modal response will be finite. Bridge responses for a typical bridge will be demonstrated in the example.

Energy harvesting

For a vibrating bridge, electromagnetic or piezoelectric energy harvesters can be used. In the following description, a cantilever piezoelectric harvester is assumed, although the mathematical expressions are valid for stack-type harvesters also. The derivations are easily extended to electromagnetic harvesters using the parameter maps given by Halvorsen (2008). The key issue for energy harvesters for highway bridges is the low frequency of the significant ambient vibration, particularly due to moving vehicles traversing the bridge. In these circumstances, electromagnetic harvesters are easier to design as coil springs may be used with magnets with a large mass to obtain low resonance frequencies. Cantilever beam harvesters, particularly where the excitation is vertical, may have problems with large static deformations due to self-weight. However, such problems may be overcome using non-linear springs and the concept of high-static–low-dynamic stiffness used for vibration absorbers and vibration isolation (Carrella et al., 2007, 2009).

Energy is harvested through base excitations and here we use a simple SDOF model for the mechanical motion of the harvester. Erturk and Inman (2008a,b, 2009) gave a more detailed model, along with correction factors for an SDOF model that accounts for distributed mass effects. This enables the analysis described here to be used in a wide range of practical applications. The SDOF model could be extended to multi-degree-of-freedom mechanical systems using a modal decomposition of the response. However, if the tip mass is large relative to the mass of the beam, then the second natural frequency will be significantly higher than the first natural frequency and the SDOF model is sufficient. This article only considers a linear model of the piezoelectric material, which allows the application of linear vibration theory.

The harvester is attached to the beam at position x, and thus, the base excitation on the harvester is the corresponding acceleration given by $\ddot{y}(x, t)$. Note that the harvester mass is assumed to be negligible compared to the mass of the bridge, and hence, the harvester mass will not affect the bridge response.

For a piezoelectric harvester, two types of simple electric circuits can be considered, namely, with and without an inductor (Adhikari et al., 2009; Erturk and Inman, 2009). Since the base excitation frequencies are likely to be very low in civil engineering applications, this will lead to a high optimum value for the inductance. Generally, such an inductance would be implemented synthetically and hence would consume power. Unless the energy harvested was significantly increased using the inductor, this circuit is unlikely to be practical.

Energy harvester without an inductor

duToit and Wardle (2007) expressed the coupled electromechanical behavior of a piezoelectric energy harvester without an inductor by the coupled linear ODEs:

$$m_\mathrm{h}\ddot{z} + c_\mathrm{h}\dot{z} + k_\mathrm{h}z - \theta V = -m_\mathrm{h}\ddot{y}(x,t) \tag{8}$$

$$\theta\dot{z} + C_\mathrm{p}\dot{V} + \frac{1}{R_\mathrm{l}}V = 0 \tag{9}$$

Equation (8) is simply Newton's equation of motion for an SDOF system, where $z(t)$ is the relative displacement of the mass, m_h. The subscript 'h' stands for the harvester; c_h and k_h are, respectively, the damping and the stiffness of the harvester. $y(x, t)$ is the base excitation for the harvester and the response of the beam at the location of the harvester is obtained from Equation (2) in the time domain or Equation (6) in the frequency domain. θ is the electromechanical coupling and the mechanical force is modeled as proportional to the voltage across the piezoceramic, $V(t)$. Equation (9) is obtained from the electrical circuit, where the voltage across the load resistance arises from the mechanical strain through the electromechanical coupling, θ, and the capacitance of the piezoceramic, C_p.

Transforming Equations (8) and (9) into the frequency domain and dividing the first equation by m_h and the second equation by C_p, we obtain:

$$\left(-\omega^2 + 2\zeta_\mathrm{h}\omega_\mathrm{h}\mathrm{i}\omega + \omega_\mathrm{h}^2\right)Z(\omega) - \frac{\theta}{m_\mathrm{h}}V(\omega) = \omega^2 Y(x,\omega) \tag{10}$$

$$\frac{\theta}{C_\mathrm{p}}\mathrm{i}\omega Z(\omega) + \left(\mathrm{i}\omega + \frac{1}{C_\mathrm{p}R_\mathrm{l}}\right)V(\omega) = 0 \tag{11}$$

where $Z(\omega)$, $V(\omega)$, and $Y(x, \omega)$ are the Fourier transforms of $z(t)$, $V(t)$, and $y(x, t)$, respectively. The mechanical natural frequency of the harvester, ω_h, and the damping factor, ζ_h, are defined as:

$$\omega_\mathrm{h} = \sqrt{\frac{k_\mathrm{h}}{m_\mathrm{h}}} \quad \text{and} \quad \zeta_\mathrm{h} = \frac{c_\mathrm{h}}{2m_\mathrm{h}\omega_\mathrm{h}}. \tag{12}$$

The solution to Equations (10) and (11) is (Adhikari et al., 2009):

$$\begin{Bmatrix} Z \\ V \end{Bmatrix} = \frac{\omega^2 Y(x,\omega)}{\Delta_1(\omega)}\begin{Bmatrix} (\mathrm{i}\omega\alpha + \omega_\mathrm{h}) \\ -\mathrm{i}\omega\alpha\theta/C_\mathrm{p} \end{Bmatrix} \tag{13}$$

where the determinant of the coefficient matrix is:

$$\Delta_1(\omega) = -\alpha i\omega^3 - (2\zeta_h\alpha + 1)\omega_h\omega^2 + \left(\alpha + \kappa^2\alpha + 2\zeta_h\right)i\omega_h^2\omega + \omega_h^3. \quad (14)$$

The non-dimensional electromechanical coupling coefficient is:

$$\kappa^2 = \frac{\theta^2}{k_h C_p}. \quad (15)$$

and the time constant of the first-order electrical system, non-dimensionalized using the natural frequency of the mechanical system, is:

$$\alpha = \omega_h C_p R_l. \quad (16)$$

Hence, the voltage in the frequency domain is given by:

$$V(\omega) = T_{h1}(\omega)Y(x,\omega) \quad (17)$$

where

$$T_{h1}(\omega) = -\frac{i}{\Delta_1}\frac{\omega^3\alpha\theta}{C_p} \quad (18)$$

represents the transfer function of the energy harvester. This simple representation will facilitate the discussion on the optimal location of the harvester in Section 4.

Energy harvester with an inductor

For this case, following Renno et al. (2009), the electrical equation becomes:

$$\theta\ddot{z} + C_p\ddot{V} + \frac{1}{R_l}\dot{V} + \frac{1}{L}V = 0 \quad (19)$$

where L is the inductance of the circuit. Proceeding as in Section 3.1 (Adhikari et al., 2009), the expression for voltage in case of a harvester with an inductor is:

$$V(\omega) = T_{h2}(\omega)Y(x,\omega) \quad (20)$$

where

$$T_{h2}(\omega) = \frac{1}{\Delta_2}\frac{\omega^4\alpha\beta\theta}{C_p} \quad (21)$$

and

$$\Delta_2(\omega) = \alpha\beta\omega^4 - (2\zeta_h\alpha + 1)\beta\omega_h i\omega^3 - \left(\alpha + \alpha\beta + 2\zeta_h\beta + \kappa^2\alpha\beta\right)\omega_h^2\omega^2 + (\beta + 2\zeta_h\alpha)\omega_h^3 i\omega + \alpha\omega_h^4. \quad (22)$$

A second non-dimensional constant has been defined as the ratio of the mechanical to electrical natural frequencies, given by:

$$\beta = \omega_h^2 L C_p. \quad (23)$$

Note that the base excitation (the beam response) is same in both Equations (17) and (20).

Estimating the total energy

The total energy from the harvester due to the vehicle passing over the bridge is obtained by integrating the instantaneous power as:

$$E_h = \int_0^\infty \frac{V(t)^2}{R_l}\mathrm{d}t \quad (24)$$

where $V(t)$ is the voltage across the load resistor, R_l. Although the force applied to the bridge is non-zero only while the vehicle is traversing the bridge, the bridge continues to vibrate and thus, the voltage from the energy harvester continues indefinitely; hence, the upper limit on the integral in Equation (24) is infinity. Alternatively, using Parseval's theorem and noting that $V(t) = 0$ for $t \leq 0$, the energy is:

$$E_h = \int_{-\infty}^{\infty} \frac{|V(\omega)|^2}{R_l}\mathrm{d}\omega \quad (25)$$

where $V(\omega)$ is the Fourier transform of $V(t)$.

Optimum harvester location and parameters

Determining the optimal locations for the harvester along the length of the bridge requires that the energy scavenged is maximized. However, this requires a formal optimization of Equation (25) and the location will depend on the vehicle, bridge, and harvester properties. In general, this is a very difficult task but some general comments may be made.

For a given bridge, the natural frequencies and mode shapes are fixed. It is very unlikely that any requirements for energy harvesting will affect the design of any future bridge. The velocity of the vehicle will vary over a large range, and for many bridges and road systems, data are available to estimate the probability density function of the vehicle velocities and also the arrival time at the bridge. The bridge design will usually try to ensure that the vehicle excitation does not correspond to a resonance frequency of the bridge. Note that we model the system as linear and so the response of the bridge and harvester (and therefore the energy and average power produced) due to multiple vehicles is easily obtained by summing the individual responses.

Most vibration energy harvesters, whether piezoelectric cantilever beams or electromagnetic systems, have one predominant resonance frequency. The passing of the vehicle over the bridge is likely to excite transients in the bridge and the predominant response is likely to correspond to the bridge modes. Thus, the harvester resonance frequency is likely to be tuned to one of the natural frequencies of the bridge. Given that the vehicle speeds are such that the lower frequencies will be excited the most, this is likely to be the fundamental bridge natural frequency.

Suppose the harvester is tuned to the j th bridge natural frequency. This means that only the bridge response corresponding to the j th mode will lead to a significant response of the harvester, assuming the bridge natural frequencies are well separated. Hence, we may approximate the voltage output from the harvester, from Equations (6), (17), and (20), as:

$$V(\omega) = T_{\rm h}(\omega) Q_j(\omega) \phi_j(x) \tag{26}$$

where $T_{\rm h}$ is used to denote either $T_{\rm h1}$ and $T_{\rm h2}$, depending on the electrical circuit employed. Thus, the total energy from the vehicle given in Equation (25) may be written as:

$$\begin{aligned} E_{\rm h} &= \int_{-\infty}^{\infty} \frac{|T_{\rm h}(\omega) Q_j(\omega) \phi_j(x)|^2}{R_{\rm l}} {\rm d}\omega \\ &= |\phi_j(x)|^2 \int_{-\infty}^{\infty} \frac{|T_{\rm h}(\omega) Q_j(\omega)|^2}{R_{\rm l}} {\rm d}\omega. \end{aligned} \tag{27}$$

From this it is clear that the optimum location for the harvester will be at the maximum of $|\phi_j(x)|$ or the antinodes for the mode shape to which the harvester is tuned. Of course, in practice, the locations of the harvesters may be specified, in which case the harvester should be tuned to a mode with a significant response at the specified location.

Numerical example

Simulation studies are performed based on the parameters specified by Elvin et al. (2006) and Yang et al. (2004) and are given in Table 1. The parameters of the bridge are taken directly from Yang et al. (2004) and the critical vehicle speed is 104.2 m/s (equivalent to 233 km/h). A typical vehicle speed of 25 m/s (90 km/h) corresponds to a fundamental excitation frequency of 0.5 Hz. The length of the beam in the harvester described by Elvin et al. (2006) has been increased so that the first natural frequency of the harvester matches the first natural frequency of the bridge (Renno et al., 2009; Ali et al., 2010). This is required to scavenge the maximum energy from the bridge vibration. To obtain the SDOF harvester model, the mass, the coupling coefficient, and the capacitance are all assumed to vary linearly with beam length. The harvester beam stiffness is assumed to be proportional to the reciprocal of length cubed. Elvin et al. (2006) selected the thickness of piezoelectric layer and the backing layer such that the maximum tip deflection was restricted to 2 mm and the strain in PVDF material was low. The value of κ may be calculated from the harvester design and this is given in Table 1; the values of α and β depend on the design of the electrical circuit and will be optimized to maximize the power available.

Table 1. Parameter values used in the simulation

Bridge		Energy harvester	
m	4800 kg/m	$m_{\rm h}$	2.500 g
ζ	0.02	ζ	0.038
E	27.5 GN/m^2	$k_{\rm h}$	0.4286 N/m
I	0.12 m^4	$\omega_{\rm h}$	2.084 Hz
L	25 m	θ	7.501 μC/m
P	12 kN	$C_{\rm p}$	2.866 nF
ω_1	2.084 Hz	κ	0.2140
$u_{\rm c}$	104.2 m/s		

The harvested power depends on the base excitation at the harvester location and therefore, increased bridge excitation provides increased harvested power. The highest excitation occurs when the excitation frequency matches a bridge's natural frequency. Physically this is unlikely, because the resonance may damage the bridge and also the corresponding critical velocity is high. Here, we consider a range of vehicle speeds within the range given by Yang et al. (2004), namely, 10–25 m/s.

Figure 2 shows the response in the frequency domain for the displacement of the bridge at different harvester locations for a vehicle velocity of $u = 25$ m/s, from Equations (6) and (7). The responses are obtained using 20 modes of the bridge. Three separate locations are considered for the harvester at one quarter ($L/4$), one third ($L/3$), and at half ($L/2$) the length of the bridge. Note that the magnitude and the frequency content of the curves change as the harvester position is changed. Since the vehicle speed is relatively low, the peak displacement responses for all these three locations are seen to be at the first natural frequency of the bridge. Note that the third mode at $\omega_3/\omega_1 = 9$ does not appear in the response at $L/3$ because this location is a node of the mode shape. Similarly, the second mode at $\omega_2/\omega_1 = 4$ does not appear in the response at $L/2$. The remaining results are for a harvester located at $L/3$. Figure 3 shows the time response of the bridge for $u = 25$ m/s calculated by the inverse FFT from the frequency domain result in Figure 2. The response may also be calculated by direct numerical integration of the equations of motion in the time domain, and the result is exactly the same. The response shows a relatively complex motion while the vehicle is on the bridge. However, once the vehicle has left the bridge, the response is essentially the damped first modal response.

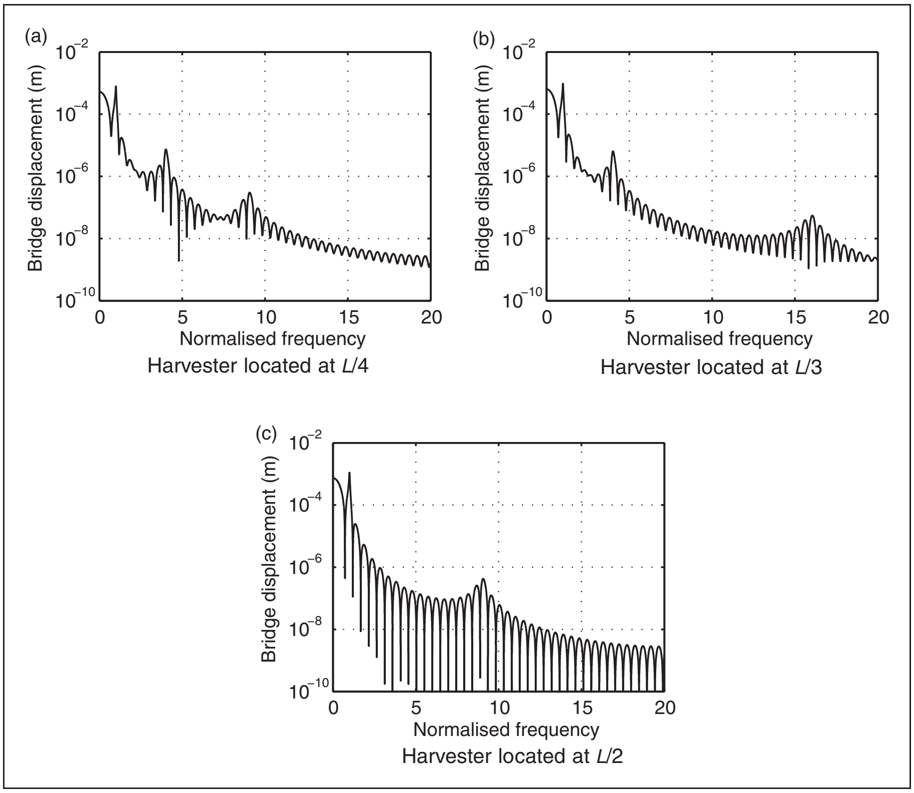

Figure 2. Frequency response of the base excitation of the harvester for different harvester locations (L/4(a), L/3(b), and L/2(c)) and a vehicle speed $u = 25$ m/s.
Note: The frequency, ω, is normalized by the first natural frequency of the bridge, ω_1.

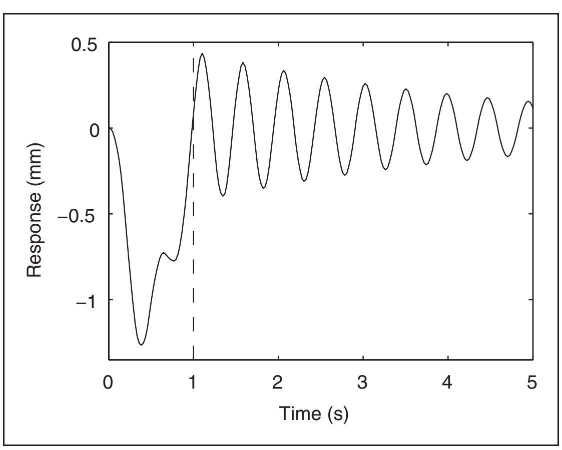

Figure 3. Time response of the bridge at location L/3 and vehicle speed $u = 25$ m/s.
Note: The vertical dashed line indicates the time when the vehicle leaves the bridge.

Figure 4 shows the low frequency response for different vehicle speeds. Except for $u = 15$ m/s, there is a distinct peak in the excitation spectrum at the first natural frequency of the bridge. There are also distinct zero responses at frequencies given, from Equation (7), by $\omega = [1\ 3\ 5\ 7\ 9\ldots]\ \pi u/L$. For $u = 15$ m/s, one of these frequencies is close to the first natural frequency of the bridge, and hence there is no resonance.

The energy generated by the harvester for different vehicle speeds may be calculated by numerical integration of Equation (25). The non-dimensional frequency increment is chosen as 7.63×10^{-5} and is sufficiently small to capture the peaks in the frequency response data. For the electrical circuit without an inductor, the load resistance must be chosen, which is equivalent to choosing a value for α (since the harvester's natural frequency is designed to be equal to the first natural frequency of the bridge and the capacitance is fixed by the choice of piezoelectric sensor). Figure 5 shows the energy generated for four vehicle speeds and a range of values of α. The energy generated is small, but it should be remembered that the harvester mass is only 2.5 g; increasing the harvester size, and therefore mass, will increase the energy produced. Clearly, there is an optimum choice of α that generates the maximum energy; this value of α is less than 1 and varies slightly with vehicle speed between 0.85 and 0.9 approximately. Suppose α is now fixed at 0.9 and the vehicle speed speed, u, varied between 10 and 25 m/s. The energy generated is shown in Figure 6; the effect is clearly

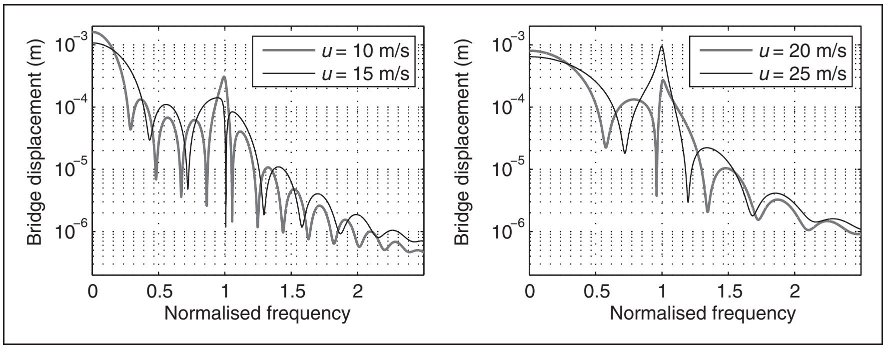

Figure 4. Frequency response of the base excitation of the harvester located at *L*/3 for different vehicle speeds, *u*. Note: The frequency, ω, is normalized by the first natural frequency of the bridge, ω_1.

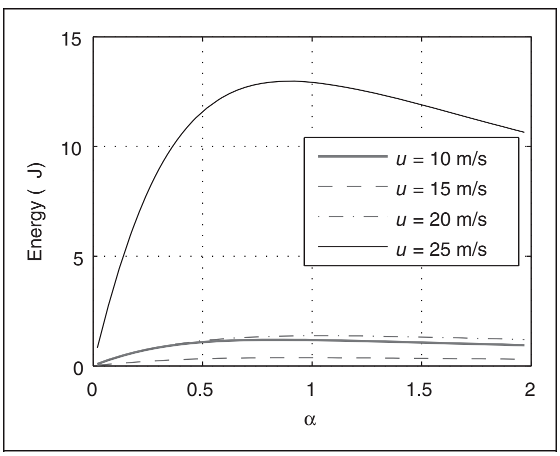

Figure 5. The variation in the energy generated by the harvester located at *L*/3 with α for a single vehicle traveling at different speeds, *u*.

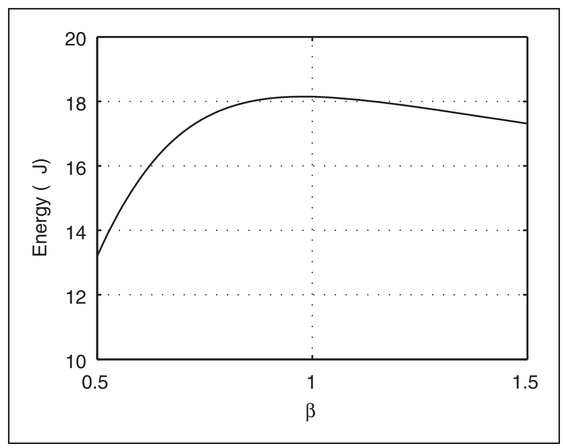

Figure 7. The variation in the energy generated by the harvester, incorporating an inductor and located at *L*/3, with β, for $\alpha = 0.9$ and $u = 25$ m/s.

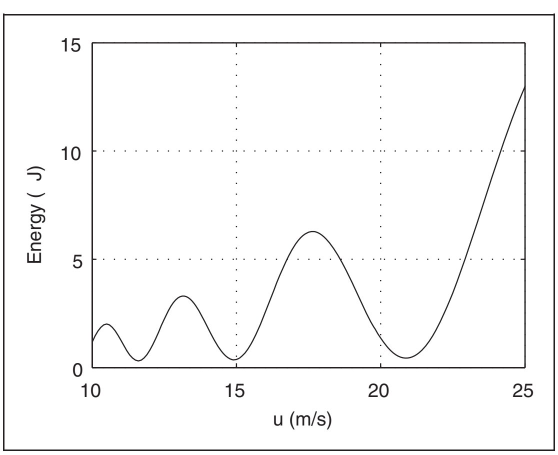

Figure 6. The variation in the energy generated by the harvester located at *L*/3 with vehicle speed for $\alpha = 0.9$.

seen of the interaction of the zeros in the bridge's response (that depend on the vehicle speed) and the first natural frequency of the bridge, as highlighted in Figure 4.

Figure 7 shows the effect of varying β on the energy generated from a harvester incorporating an inductor. For this case, the load resistance has been fixed so that $\alpha = 0.9$ and a single vehicle speed of $u = 25$ m/s has been considered. The maximum energy generated occurs for a value of β just below 1; note that $\beta = 1$ means that the mechanical and electrical natural frequencies of the harvester are equal. The maximum energy generated has been increased over the electrical circuit without an inductor by approximately 40%. If the inductor were implemented synthetically (and hence would consume power), then the small increase in energy generated would probably be insufficient to make the inductor circuit practical.

Conclusion

The increasing need for wireless SHM increases the requirements to power these devices. An environmental friendly possibility is to harvest energy from the host structure to generate power for the wireless devices. This article assumes a single piezoelectric energy harvester embedded inside the bridge deck and considers two types of electrical circuits. An analytical formulation is given for the energy generated by the harvester for a single vehicle of a given mass crossing at a known speed, and this is easily extended for a general finite element model of the bridge. The harvester is tuned to the first resonance frequency of the bridge, as this produces the highest energy. The generated energy varies significantly with vehicle speed and mass. This article provides a preliminary parametric study of an energy harvester on a highway bridge excited by a simple vehicle model, but provides valuable insights. For example, it is unlikely that the resonant electrical circuit including an inductor will be viable for the low excitation frequencies encountered in this application. More advanced vehicle models may be incorporated into the formulation described and may provide a wider range of response frequencies to harvest energy.

The extension of the analysis described to multiple vehicles is very straight-forward since the equations of motion are linear. If vehicles of equal mass arrived at the bridge at regularly spaced intervals with identical speeds, then the energy results in this article are easily converted into average power. More realistically, traffic statistics will determine probability density functions for the time of arrival of vehicles at the bridge, and also their mass and speed. Using the analysis described in this article, these statistics are easily combined using a Monte Carlo analysis into a probability density function for the average power generated. Clearly, this average power will be a function of the traffic flow and hence indirectly, the day of the week and the time. However, such information would allow an energy management system to be designed, which would include energy storage in some form and would account for the requirements of the health monitoring or other system to be powered.

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