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Experimental case studies for uncertainty quantification in structural dynamics

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ABSTRACT

The consideration of uncertainties in numerical models to obtain the probabilistic descriptions of vibration response is becoming more desirable for industrial-scale finite element models. Broadly speaking, there are two aspects to this problem. The first is the quantification of parametric and non-parametric uncertainties associated with the model and the second is the propagation of uncertainties through the model. While the methods of uncertainty propagation have been extensively researched in the past three decades (e.g., the stochastic finite element method), only relatively recently has quantification been considered seriously. This paper considers uncertainty quantification with the aim of gaining more insight into the nature of uncertainties in medium- and high-frequency vibration problems. This paper describes the setup and results from two experimental studies that may be used for this purpose. The first experimental work described in this paper uses a fixed–fixed beam with 12 masses placed at random locations. The total 'random mass' is about 2% of the total mass of the beam and this experiment simulates 'random errors' in the mass matrix. The second experiment involves a cantilever plate with 10 randomly placed spring-mass oscillators. The oscillating mass of each of the 10 oscillators is about 1% of the mass of the plate. One hundred nominally identical dynamical systems are created and individually tested for each experiment. The probabilistic characteristics of the frequency response functions are discussed in the low, medium and high frequency ranges. The variability in the amplitude of the measured frequency response functions is compared with numerical Monte Carlo simulation results. The data obtained in these experiments may be useful for the validation of uncertainty quantification and propagation methods in structural dynamics.

1. Introduction

Finite element codes implementing physics-based models are used extensively for the dynamic analysis of complex systems. Laboratory-based controlled tests are often performed to gain insight into some specific physical aspects of a problem. Such tests can indeed lead to new physical laws improving the computational models. Test data can also be used to calibrate a known model. However, neither of these activities may be enough to produce a credible numerical tool because of several types of uncertainties which exist in the physics-based computational framework. Such uncertainties include, but are not limited to (a) parameter uncertainty (e.g, uncertainty in geometric parameters, friction coefficient, strength of the materials involved); (b) model uncertainty (arising from the lack of scientific knowledge about the model which is *a priori* unknown); (c) experimental error (uncertain and unknown errors percolate into the model when they are calibrated against experimental results). These uncertainties must be assessed and managed for credible computational predictions.

The predictions from high resolution numerical models may sometimes exhibit significant differences with the results from physical experiments due to uncertainty. When substantial statistical information exists, the theory of probability and stochastic processes offer a rich mathematical framework to represent such uncertainties. In a probabilistic setting, the data (parameter) uncertainty associated with the system parameters, such as the geometric properties and constitutive relations (i.e. Young's

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modulus, mass density, Poisson's ratio, damping coefficients), can be modelled as random variables or stochastic processes using the so-called parametric approach. These uncertainties can be quantified and propagated, for example, using the stochastic finite element method [1–14].

Recently, the uncertainty due to modelling error has received attention as this is crucial for model validation [15–23]. The model uncertainty problem poses serious challenges as the parameters contributing to the modelling errors are not available *a priori* and therefore precludes the application of a parametric approach to address such issues. Model uncertainties do not explicitly depend on the system parameters. For example, there can be unquantified errors associated with the equation of motion (linear or non-linear), in the damping model (viscous or non-viscous [24,25]), in the model of structural joints. The model uncertainty may be tackled by the so-called non-parametric method pioneered by Soize [26–28] and adopted by others [29–34].

Uncertainties associated with a variable can be characterised using the probabilistic approach or possibilistic approaches based on interval algebra, convex sets or Fuzzy sets. In this paper the probabilistic approach has been adopted. The equation of motion of a damped n-degree-of-freedom linear structural dynamic system can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \quad (1)$$

where $\mathbf{M} \in \mathbb{R}_{n,n}$, $\mathbf{C} \in \mathbb{R}_{n,n}$ and $\mathbf{K} \in \mathbb{R}_{n,n}$ are the mass, damping and stiffness matrices respectively. The importance of considering parametric and/or non-parametric uncertainty also depends on the frequency of excitation. For example, in high-frequency vibration the wavelengths of the vibration modes become very small. As a result the vibration response can be very sensitive to the small details of the system. In general, three different approaches are currently available to analyse stochastic structural dynamic systems across the frequency range:

- *Low-frequency vibration problems*: Stochastic Finite Element Method [1–14] (SFEM)—considers parametric uncertainties in detail;
- *High frequency vibration problems*: Statistical Energy Analysis [35–37] (SEA)—does not consider parametric uncertainties in detail;
- *Mid-frequency vibration problems* [38–42]: both parametric and non-parametric uncertainties need to be considered.

The majority of the studies reported in probabilistic mechanics are based on analytical or simulation methods. Often simulation based methods are used to validate approximate but relatively fast prediction tools (such as perturbation-based methods). Experimental results are rare because of difficulties such as (a) generating nominally identical samples of a structural system, (b) the resources and effort involved in testing a large number of samples, (c) the repetitive nature of the experimental procedure and (d) ensuring that different samples are tested in exactly the same way so that no uncertainty arises due to the measurement process. In spite of these difficulties some authors have conducted experimental investigations on random dynamical systems. For structure-borne noise, Kompella and Bernhard [43] measured 57 frequency response functions at driver microphones for different pickup trucks. Fahy [44, page 275] reported measurements of FRFs on 41 nominally identical beer cans. Both of these experiments show variability in nominally identical engineered systems. Friswell et al. [45] reported two experiments where random systems were 'created' in the laboratory for the purpose of model validation. The first experiment used a randomly moving mass on a free-free beam and the second experiment involved a copper pipe with uncertain internal pressure. Fifty nominally identical random samples were created and tested for both experiments.

Table 1
Material and geometric properties of the beam considered for the experiment.

Beam properties	Numerical values
Length (L)	1200 mm
Width (b)	40.06 mm
Thickness (t_h)	2.05 mm
Mass density (ρ)	7800 kg/m^3
Young's modulus (E)	2.0×10^5 MPa
Cross sectional area ($a = bt_h$)	8.212×10^{-5} m^2
Moment of inertia ($I = 1/12bt_h^3$)	2.876×10^{-11} m^4
Mass per unit length (ρ_l)	0.641 kg/m
Bending rigidity (EI)	5.752 N m^2
Total weight	0.7687 kg

This paper describes two experimental studies that may be used to test methods of uncertainty quantification across the frequency range. The tests are closely controlled and the uncertainty can be considered to be 'known' for all practical purposes. This allows one to model uncertainty, propagate it through dynamical models and compare the results with this experimentally obtained data. The first experiment described in this paper uses a fixed–fixed beam with 12 masses placed at random locations. The total *random mass* is about 2% of the total mass of the beam and this experiment simulates *random errors* in the mass matrix. One hundred nominally identical dynamical systems are created and individually tested in the Bristol Laboratory for Advanced Dynamic Engineering (BLADE). The model of the beam and experimental setup are described in Section 2.1. The experimental method to test one hundred nominally identical structures is discussed in Section 2.2. In Section 2.3 the probabilistic characteristics of the amplitude of the measured frequency response functions are discussed in the low, medium and high frequency ranges. In Section 2.4 the experimental system with random mass distribution is numerically simulated using Euler–Bernoulli beam theory and Monte Carlo simulation. In Section 2.5 the mean and standard deviation of the amplitude of the experimentally measured frequency response functions are compared with Monte Carlo simulation results. The model of the cantilever plate and the experimental setup are described in Section 3.1. The experimental method to test one hundred nominally identical systems is discussed in Section 3.2. In Section 3.3 the probabilistic characteristics of the amplitude of the measured frequency response functions are discussed in the low-, medium- and high-frequency ranges. In Section 3.4 the experimental system with random mass distribution is numerically simulated using the thin plate theory and Monte Carlo simulation. In Section 3.5 the mean and standard deviation of the amplitude of the experimentally measured frequency response functions are compared with Monte Carlo simulation results. The key results and the contributions of this work are discussed Section 4. The data presented here are available on the world wide web for research purposes. The web address is http://engweb.swan.ac.uk/~adhikaris/uq/. This data may be used to validate different uncertainty quantification and propagation methods in structural dynamics.

2. The beam experiment

2.1. System model and experimental setup

A steel beam with uniform rectangular cross-section is used for the experiment. The physical and geometrical properties of the steel beam are shown in Table 1.

The beam is actually a 1.5 m long ruler made of steel. The use of a ruler ensures that the masses may be easily placed at predetermined locations. The ruler is clamped between 0.05 m and 1.25 m so that the effective length of the vibrating beam is 1.2 m. The overall experimental setup is shown in Fig. 1.

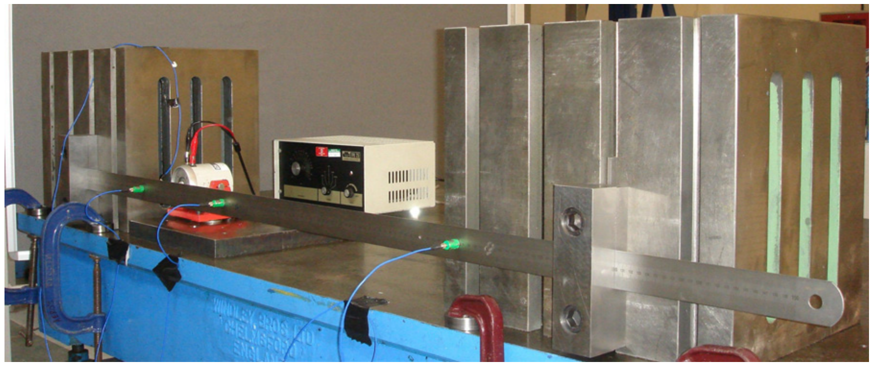
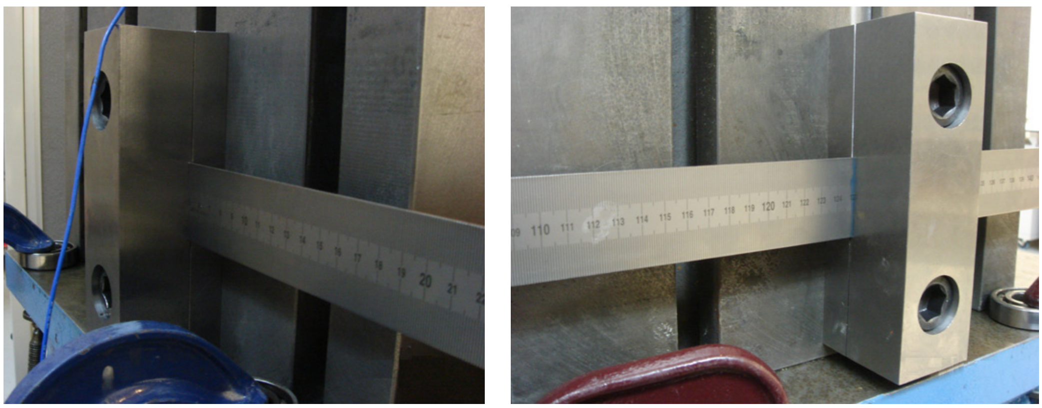


Fig. 1. The test rig for the fixed–fixed beam.



(a) Left end. (b) Right end.

Fig. 2. The clamping arrangements at the two ends of the beam. The clamping arrangement is aimed at providing fixed–fixed boundary conditions.

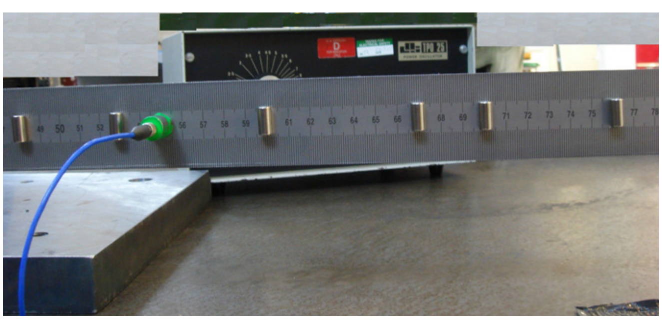


Fig. 3. The attached masses (magnets) at random locations. In total 12 masses, each weighting 2 g, are used.

The end clamps are bolted to two heavy steel blocks, which in turn are fixed to a rigid table with bolts as shown in Fig. 2. The clamping arrangements are aimed at providing fixed–fixed boundary conditions.

The edges of both the solid metal blocks are sharpened to ensure that no rotation is allowed beyond the 'true end' of the beam. It should be emphasised that it is, in general, extremely difficult to ensure there is no residual stress in a clamped or any other constrained system. Due to the fixed nature of the supports and flexibility of the beam, our initial tests show that the beam was in compression and was buckling in the first mode (the first natural frequency turned out to be close to zero and significantly lower than predicted by the Euler–Bernoulli beam theory). The length between the supports was adjusted to minimise the compression or tension in the beam. Because the supports were not disturbed during the entire test spanning 100 realisations, we expect that the residual stress did not change from sample to sample.

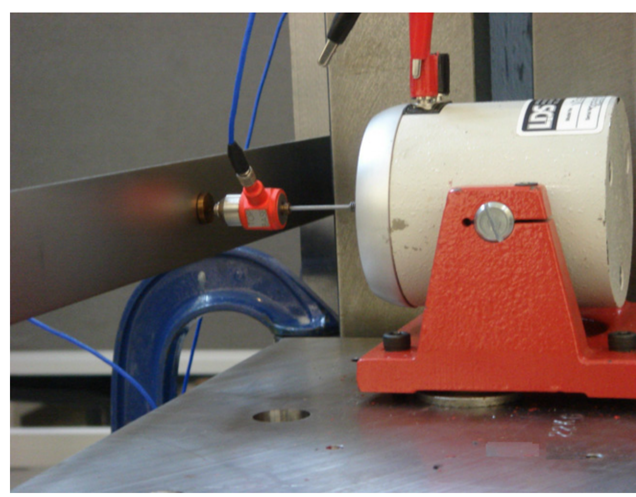


Fig. 4. The shaker is used as an impulse hammer which in turn is controlled via Simulink™and dSpace™. A hard steel tip was used and small brass plate weighting 2 g is attached to the beam to take the impact from the shaker.

Twelve equal masses are used to simulate a randomly varying mass distribution. The masses are actually magnets so that they can be easily attached at any location on the steel beam. These magnets are cylindrical in shape, with a length of 12.0 mm and a diameter of 6.0 mm. Some of the attached masses for a sample realisation are shown in Fig. 3.

Each mass weighs 2 g so that the total variable mass is 1.6% of the mass of the beam. The location of the 12 masses are assumed to be between 0.2 m and 1.0 m from the left end of the beam. A uniform distribution with 100 samples is used to generate the

Table 2
The details of the accelerometers and the force transducer for the beam experiment.

Role	Model & Serial number	Position from the left end	Channel	Sensitivity
Sensor (accelerometer)	PCB 333M07 SN 25948	23 cm (Point 1)	1	98.8 mV/g
Sensor (accelerometer)	PCB 333M07 SN 26018	50 cm (Point 2)	2	101.2 mV/g
Sensor (accelerometer)	PCB 333M07 SN 25942	102 cm (Point 3)	3	97.6 mV/g
Actuator (force transducer)	PCB 208C03 21487	50 cm (Point 2)	4	2.24 mV/N

mass locations. The mean and the standard deviations of the mass locations are given by

$$\bar{\mathbf{x}}_m = [0.2709, 0.3390, 0.3972, 0.4590, 0.5215, 0.5769, 0.6398, 0.6979, 0.7544, 0.8140, 0.8757, 0.9387]\ \text{m} \qquad (2)$$

and

$$\sigma_{\mathbf{x}_m} = [0.0571, 0.0906, 0.1043, 0.1034, 0.1073, 0.1030, 0.1029, 0.1021, 0.0917, 0.0837, 0.0699, 0.0530]\ \text{m}. \qquad (3)$$

2.2. Experimental methodology

Experimental modal analysis [46–48] is used in this work. The three main components of the implemented experimental technique are (a) the excitation of the structure, (b) the sensing of the response, and (c) the data acquisition and processing. In this experiment a shaker was used (the make, model no. and serial no. are LDS, V201, and 92358.3, respectively) to act as an impulse hammer. The usual manual impact hammer was not used because of the difficulty in ensuring the impact occurs at exactly at the same location with the same force for every sample run. The shaker generates impulses at a pulse interval of 20 s and a pulse width of 0.01 s. Using the shaker in this way eliminates, as far as possible, any uncertainties arising from the input forces. This innovative experimental technique is designed to ensure that the resulting uncertainty in the response arises purely due to the random locations of the attached masses. Fig. 4 shows the arrangement of the shaker.

(a) Response across the frequency range.

(b) Low-frequency response.

(c) Medium-frequency response.

(d) High-frequency response.

Fig. 5. Experimentally measured amplitude of the FRF of the beam at point 1 (23 cm from the left end) with 12 randomly placed masses. 100 random FRFs, together with the FRF of the baseline system (- -) ensemble mean (-), 5% (-.-) and 95% (...) probability lines are shown.