FLC-MR DAMPER CONTROL OF STAY CABLE NONLINEAR VIBRATION

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Abstract

Stay cables are the main load carrying members of cable-stayed bridges, and large oscillations in the cables may cause catastrophic failure of the bridge. Therefore, modeling and controlling the vibration of cables under dynamic loads have become essential design requirements. The paper develops a fuzzy logic rule base to monitor the voltage of magnetorheological dampers to minimize stay cable responses. Cable nonlinearity, uncertainty with reference to magnitude and frequency content of dynamic loads, possibility of sensor saturation, limitations in actuator capacity and saturation have been given due consideration in developing control algorithms. Fuzzy logic control (FLC) with MR damper provides inherent robustness to the control mechanism and is easy to implement in real structures. The proposed control approach and classical control approaches are compared for a cable oscillation example.

Key words: FLC, Cable Vibration, Nonlinear Analysis

INTRODUCTION

Stay cables are important structural elements for many civil engineering structures and are extensively used in applications ranging from long span bridges to marine cables, transmission towers to temporary structures. Their importance in civil structures is increasing due to their immense tensile strength. They are the main load carrying members in long-span bridges and temporary structures. Unexpected large oscillation in stays of cable stayed bridges and vertical hangers of suspension and arch bridges are due to movement in their support points, i.e., tower top and deck connection. Low inherent damping of cables should be supported by control measures to mitigate such excessive vibration levels. In the past decade, the cable vibration control techniques by means of various passive countermeasures have been extensively investigated [Fujino 1994; Warnitchai 1995]. Performance of passive dampers increases with increase in distance between point of installation and support. Present day cable stayed bridges are several thousands metres long making each cable length nearly 500 metres, therefore installation of passive dampers becomes difficult. Numerous researchers have studied active vibration control of cables through the application of transverse force and axial stiffness or tension [Susumpow 1995]. Ni [2002] studied the neuro-control of inclined sagged cables using semi-active MR dampers. Johnson [2000] investigated the semi-active vibration control of a cable using MR dampers based on the clipped control strategy. Clipped optimal algorithm swaps the damper voltage between zero and its maximum value, thereby, does not make use of the full voltage range. Furthermore, Linear Quadratic Gaussian (LQG) control needs accurate analysis and state estimation, involving huge computational time. Therefore, nonlinearity and uncertainty in analysis may drive the system to instability. Moreover, the time lag between analysis and actual implementation makes the overall system sub-optimal. The present study uses a fuzzy rule base to control the voltage of the MR damper. Thereby, one can not only make use of the full voltage range available, but also get rid of the slight uncertainty present in modelling cable parameters and input excitation. Most studies on cable vibration control either analyzed a linear model of the stay cable or studied wind-induced vibration

[Johnson 2000]. In the present study, the nonlinear response characteristics of a stay cable controlled by FLC driven MR damper is analyzed based on an accurate cable model. FLC has inherent robustness under nonlinear model, and MR dampers are fail-safe, as they need only a small voltage to operate and even controls in the absence of voltage supply. Finally the paper provides a comparison of the proposed control method with other control techniques.

CABLE NON-LINEAR ANALYSIS

Cables remain attached to structural system like the bridge deck or supports at tower top, and experience support motion during either the movement of the deck or tower or both. Therefore, cables in cable-stayed bridges should also be analysed to account for support motion. In this section motion of a single cable is investigated where the effects of global vibration is taken into account as motions at cable supports i.e. anchorages.

Cable equation of motion

Considering an uniform cable (Fig.1) of chord length *'L'*, area of cross-section *'A'*, modulus of elasticity *'E'* and mass per unit length '*m*', with small depth to span ratio. Thus, the static configuration of the cable can be assumed to be parabolic in the gravity plane. The differential equations for the static configuration of a parabolic cable are given by [Irvine 1981]

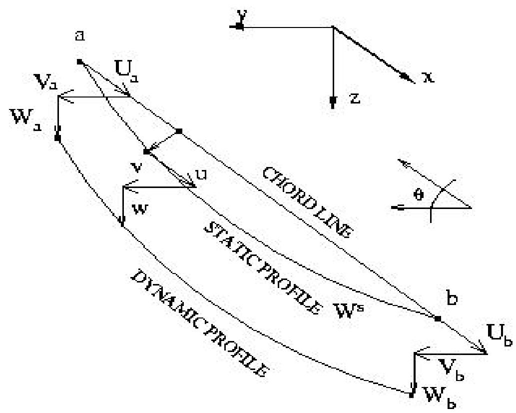

Figure 1: Typical Stay Cable With Support Motions at 'a' & 'b'.

$$H\frac{d^2w^s}{dx^2}=-mg \tag{1}$$

Where *'H'* is the static horizontal tension, *'x'* is the longitudinal coordinates of the cable system. Fig.1 shows a stay cable with supports at different heights making an angle *'θ'* with the horizontal axis. *'u', 'v'*, and *'w'* are the displacements along *'x', 'y'*, and *'z'* axes respectively. Static and dynamic profile of the cable is shown with support motions at support *'a'* and *'b'*. Superscript '*s*' refers to the static equilibrium state. The dynamic equation of motion for the cable has been found using force equilibrium in three coordinate directions [Irvine 1981].

$$\frac{\partial}{\partial s}\left[(T+\tau)\frac{\partial}{\partial s}\left(u^s+u\right)\right]+X=m\frac{\partial^2 u}{\partial t^2}\ ;$$
$$\frac{\partial}{\partial s}\left[(T+\tau)\frac{\partial}{\partial s}\left(v^s+v\right)\right]+Y=m\frac{\partial^2 v}{\partial t^2}\ ;\quad \frac{\partial}{\partial s}\left[(T+\tau)\frac{\partial}{\partial s}\left(w^s+w\right)\right]+Z=m\frac{\partial^2 w}{\partial t^2} \tag{2a,b,c}$$

Where, *'X', 'Y', 'Z'* are external forces along *'x', 'y', 'z'* axis. Assuming that the cable deforms in the elastic range, the dynamic tension 'τ' can be separated from static tension '*T*' and can be given as.

$$\tau(x,t)=EA\varepsilon(x,t)\ ;\qquad \varepsilon(x,t)=\frac{\partial u}{\partial x}+\frac{\partial w^s}{\partial x}\frac{\partial w}{\partial x}+\frac{1}{2}\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2+\left(\frac{\partial w}{\partial x}\right)^2\right] \tag{3}$$

Where *ε(x,t)* is the Green-Lagrange strain. The non-linear strain displacement relation makes a cable system nonlinear in nature. The non-homogeneous boundary conditions arising due to support motion are;

$$u(0,t)=u_a(t),\quad v(0,t)=v_a(t),\quad w(0,t)=w_a(t).\quad u(L,t)=u_b(t),\quad v(L,t)=v_b(t),\quad w(L,t)=w_b(t). \tag{4}$$

Cable Analysis with support motion

To analyse the cable under support motion the total time dependent displacements are separated into two parts, quasi-static and modal motions (for details see Ali 2005). Since, longitudinal motion of cables is less in comparison to other directions, the axial inertia force is very small and can be omitted. The axial distributed force X is assumed to be zero. After that, the modal equation with homogeneous boundary condition is solved using standard Galerkin approach. The final nonlinear differential equations of motions of stay cable are obtained as:

$$m_{yn}(\ddot{y}_n + 2\xi_{yn}\dot{y}_n + \omega_{yn}^2 y_n) + \sum_k v_{nk}\left(y_k^2 + z_k^2\right)y_n + \sum_k 2\beta_k z_k y_n + 2\eta_n(u_b - u_a)y_n + \zeta_n\left(\ddot{v}_a + (-1)^{n+1}\ddot{v}_b\right) = F_{yn} \quad (5)$$

$$\begin{aligned} m_{zn}(\ddot{z}_n + 2\xi_{zn}\dot{z}_n + \omega_{zn}^2 z_n) + \sum_k v_{nk}\left(y_k^2 + z_k^2\right)z_n + \sum_k 2\beta_{nk} z_k z_n + \sum_k 2\beta_{kn}\left(y_k^2 + z_k^2\right) \\ + 2\eta_n(u_b - u_a)z_n + \zeta_n\left(\ddot{w}_a + (-1)^{n+1}\ddot{w}_b\right) + \alpha_n(\ddot{u}_b - \ddot{u}_a) = F_{zn} \end{aligned} \quad (6)$$

The values of parameters used are given in Eq.7, where, subscript *'n'* denotes parameters corresponding to the n^{th} mode shape of the motion, i.e. $n = 1,2,3, \ldots$,.

$$\begin{aligned} & m_{yn} = m_{zn} = \tfrac{1}{2}mL;\ m_{yn}\omega_{yn}^2 = \frac{H\pi^2 n^2}{2L};\ m_{zn}\omega_{zn}^2 = \frac{H\pi^2 n^2}{2L}(1+k_n);\ k_n = \left(\frac{2\lambda^2}{\pi^4 n^4}\right)\left(1+(-1)^{n+1}\right)^2; \\ & v_{nk} = \frac{EA\pi^4 n^2 k^2}{8L^3};\ \beta_{nk} = \left(\frac{EA^2\pi\gamma n^2}{4LH}\right)\left(\frac{1+(-1)^{k+1}}{k}\right);\ \eta_n = \frac{E_q A\pi^2 n^2}{4L^2};\ \varsigma_n = \frac{mL}{n\pi};\ E_q = \frac{1}{1+\lambda^2/12}E; \\ & \alpha_n = \left(\frac{mL}{\pi^3 n^3}\right)\left(\frac{E_q\gamma L}{(H/A)^2}\right)\left(1+(-1)^{n+1}\right);\ F_{yn} = \int_0^L Y\phi_n\,dx;\ F_{zn} = \int_0^L Z\psi_n\,dx;\ \lambda^2 = \frac{EA}{H}\left(\frac{\gamma LA}{H}\right)^2;\ \gamma = mg\cos(\theta); \end{aligned} \quad (7)$$

Eq.s 5, 6 show that the cable dynamic equation of motion is highly nonlinear and coupled.

CABLE VIBRATION CONTROL

The excessive vibration in cables under earthquake or wind induced excitations results in severe damage or even catastrophic failure of long-span bridges. Therefore, cable vibration control is an important issue. The damper in a stay cable should be located within 0.05L from a support for aesthetic reasons. Optimal damper location is at 0.02L from the anchored support (Ali 2005).

Passive Viscous Fluid Damper

Widely used passive system for cable vibration control is the viscous fluid damper. The damper force is given by

$$f_d(t) = C_d\left|\dot{x}_d\right|^{\alpha_{vc}}\operatorname{sgn}(\dot{x}_d). \quad (8)$$

Where, $\dot{x}_d$ is the velocity across the damper location in the cable, and C_d is an experimentally determined damping coefficient, and α, is a real positive exponent with typical values in the range of 0.35–1 for seismic applications [Agrawal 2003]. Eq. 8 represents a linear viscous damper at '$\alpha=1$'. Nonlinear viscous dampers have become popular recently because of their nonlinear force-velocity relationships and ability to limit the peak damper forces at large velocities while providing sufficient supplemental damping. This makes the nonlinear damper efficient for near field earthquakes.

Magneto-rheological Dampers

When exposed to magnetic field, MR fluids change from free flowing, linear viscous Newtonian fluid to a semi-solid Bingham fluid with controllable yield strength in milliseconds. Spencer [1997] has reported both parametric and nonparametric models for modeling MR Dampers. The most commonly used model for MR dampers is the Bouc-Wen model. The present study uses 10 volts, 50 watt MR damper with a capacity of providing 1000 kN force.

$$f_d = c_0\dot{x} + \alpha z \tag{9}$$

The internal dynamics of the damper system is given by Eq.10. In MR damper, the damper force cannot be altered directly, therefore a clipped optimal algorithm is proposed by Dyke [1996], where, the voltage input to the cable is changed based on the control proposed by LQG control.

$$z = -\gamma\left|(\dot{x})\right| z|z|^{n-1} - \beta(\dot{x})|z|^n + A_{mr}(\dot{x}) \ ; \quad \alpha = \alpha_a + \alpha_b u; \qquad c_0 = c_{0a} + c_{0b}u \quad ; \qquad \dot{u} = -\eta(u - v) \ ; \tag{10}$$

Fuzzy Logic Driven MR Damper Voltage

The performance of clipped optimal algorithm fully depends on the accuracy of the model, as it decides the damper voltage to be supplied based on the difference of LQG proposed control force and force applied by the damper at current state [Dyke 1996]. Thus not only does it increase computation but also provide control force which is not optimal to the system. The FLC has been designed using five membership functions for each of the input variable (acceleration and velocity at mid span) and four for output variable (damper voltage) for finer input-output mapping. The subsets are NL = negative large, NE = negative, ZE = zero, PO = positive, and PL = positive large, PS = positive small (Fig.2). Generalized bell-shaped membership functions have been used for the FLC. The choice of a velocity and an acceleration component for feedback can be explained in the context of the state of the system in the fundamental mode of vibration. These feedback components help in generating the initial inference rule base (e.g., if velocity is zero and acceleration is high, the structure is at its extreme position and control action is not needed because it is going to return to its neutral position due to the restoring force, therefore voltage is zero). Again, if velocity and acceleration are of the same sign, the structure is returning to its neutral position due to its restoring force, and, if the acceleration and velocity are of opposite sign, then the structure is moving toward its extreme position and accordingly the control action should be applied. The adopted inference rules in this study are shown in Table-1.

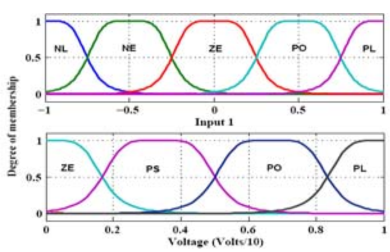

Figure 2: Input Output Membership function

Table: 01 Inference Rules for FLC

Velocity		Acceleration				
		NL	**NE**	**ZE**	**PO**	**PL**
	NL	PL	PO	PS	PS	ZE
	NE	PO	PS	ZE	ZE	ZE
	ZE	PS	ZE	ZE	ZE	PS
	PO	ZE	ZE	ZE	PS	PO
	PL	ZE	PS	PS	PO	PL

NUMERICAL RESULTS

A test cable for analysis and control is taken from [Susumpow, 1995]. The parameters of the cable and MR damper used are given in Table 2. The cable differential equations are solved using ten sine terms. Vibration analysis is carried out for El Centro Earthquake excitation at left support, right support assumed to be fixed. The damper is supposed to be located at its optimal location i.e. 0.02L [Ali, 2005] from the left support. Four types of control strategies are studied, viz., passive linear viscous damping (PLVC) (case I); passive nonlinear viscous damping (PNLVC) (case II), semi active MR damper control using clipped optimal approach (COMR) (case III) and FLC driven MR damper (FLCMRD) (case IV). For the nonlinear viscous damper, test runs were done for different α_{vc}.

Table: 02 Cable and Damper Parameters

Sym	Values	Sym	Values
L	205 m	C_{0a}	4.40 Nsec/cm
A	7.2e-3 m2	C_{0b}	44 Nsec/(cm V)
m	60.1992	n	1
E	1.962e11 N/m2	A_{mr}	1.2
H	2932560 N	γ	3 cm^{-1}
θ	38.7*π/180	β	3 cm^{-1}
α_a	1.087e5 N/cm	η	50 sec^{-1}
α_b	4.961e5 N/(cmV)	v	0-10 V

Table: 03 Comparison of Different Control Methods

	Peak Disp (m)	Normed Disp (m)	Peak Force (N)	Peak Voltage (V)
Uncont	0.6752	43.7066	--	--
PLVC	0.3001	27.2046	2.7e5	--
PNLVC	0.3220	33.7446	2.9e5	--
COMR	0.5882	28.7701	4.9e5	10
FLCMR	0.3131	28.8682	2.26e5	6.7871

Finally, $\alpha_{vc}=0.6$ is chosen for comparison. Viscous damper constant is taken as $C_d=1e6$, for both linear and nonlinear case. For case III acceleration feedback is taken at the point of location of damper. Kalman estimation technique is used to estimate unknown states and to control the process and measurement noise. FLC is proposed based on acceleration and velocity feedback and voltage as output. For case III and IV, A/D and D/A converters with 16-bit precision and a span of ±10 volts are used. Sensor sensitivity is taken as 1V=10g and actuator capacity is assumed as 1000 kN. Sensor and actuator dynamics are not considered as natural frequencies of sensors and actuators are assumed to be much higher than the cable dominant frequency range. All four cases are compared in Table 3.

Time histories of displacement at damper location, normed displacement over the span at each time instance and force time history for case I and case IV are shown in Fig.3. It is evident from the figure that FLC MR damper provides similar control as PLVC, but force given to the system is less. Fig.4 shows the normed response of cable over time along the span of the cable. Fig.5 shows the voltage required for FLC based MR damper control. It is evident that FLC has achieved it goal of getting all voltages in the range of damper. Thereby, providing better robustness than clipped optimal algorithm. Also the voltage never reaches its maximum thereby, avoids actuator saturation.

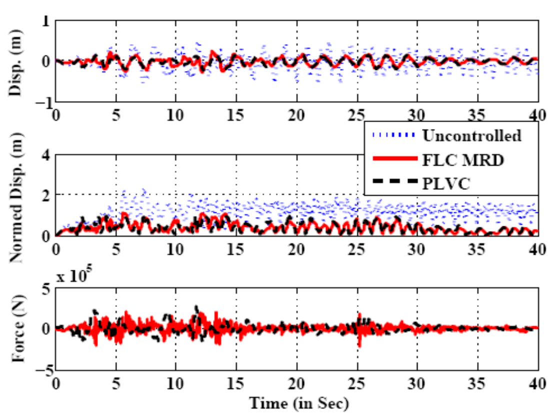

Figure 3: Time Histories

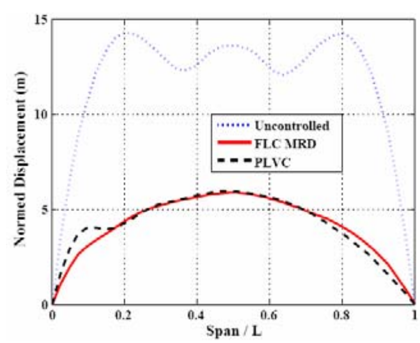

Figure 4: Normed response along span

Stability of proposed FLC based MR damper control is shown by taking worst-case initial condition, where both velocity and acceleration are given its maximum values. Fig.7 shows how the time histories of velocity and acceleration.

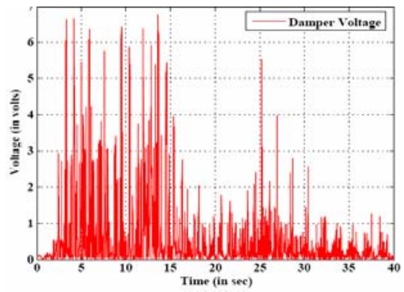

Figure 5: Voltage Time history

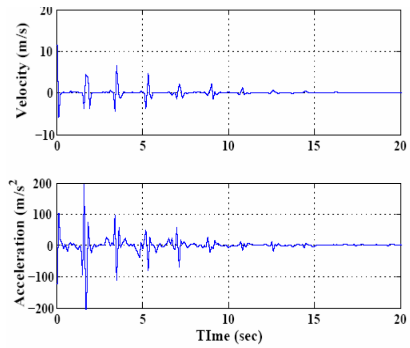

Figure 6: Velocity and Acceleration Time history

CONCLUSION

From the vibration analysis and FLC based control of the stay cable the following conclusions may be reached:

1. Passive dampers located on the cable are quite effective in limiting the vibration amplitudes. But increase in length of cable reduces their performance as the optimal location (0.02L) of the damper placement becomes inaccessible.
2. FLC based voltage switching of MR damper voltage provides better control than clipped optimal control as it varies the voltage over the full range.
3. FLC along with MR damper provides inherent robustness to the system.
4. Semi active MR dampers are not effective when deployed individually but may offer an attractive alternative to active systems when used along with passive systems as a hybrid system.

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