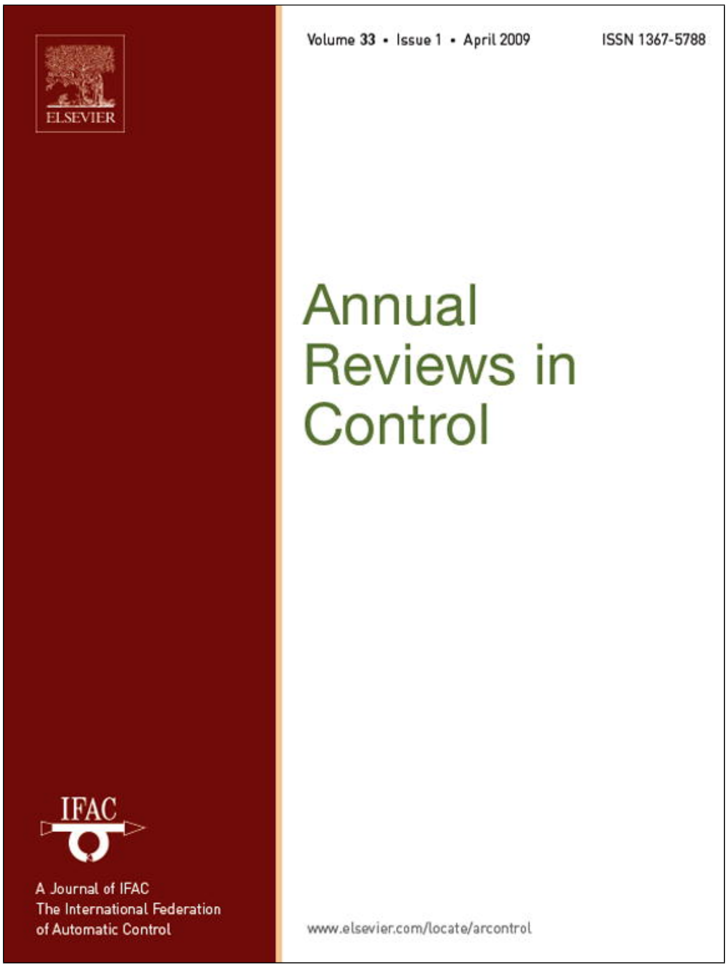
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An account of chronological developments in control of distributed parameter systems

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ABSTRACT

Control systems arising in many engineering fields are often of distributed parameter type, which are modeled by partial differential equations. Decades of research have lead to a great deal of literature on distributed parameter systems scattered in a wide spectrum. Extensions of popular finite-dimensional techniques to infinite-dimensional systems as well as innovative infinite-dimensional specific control design approaches have been proposed. A comprehensive account of all the developments would probably require several volumes and is perhaps a very difficult task. In this paper, however, an attempt has been made to give a brief yet reasonably representative account of many of these developments in a chronological order. To make it accessible to a wide audience, mathematical descriptions have been completely avoided with the assumption that an interested reader can always find the mathematical details in the relevant references.

1. Introduction

Distributed parameter systems (DPSs) are an established research area in control which traces its roots back to the 1960s. To adequately describe the distributed nature of the system one necessarily needs to use partial differential equation (PDE) models (Curtain, 2003). The general objectives behind the control of DPS are more or less the same as those for lumped parameter systems like stability, optimality, etc. In addition, it also presents certain unique challenges like satisfying the spatial boundary conditions, non-collocated, collocated control design (assuring separate locations for controllers and actuators), etc. Many innovative ideas have been put into design and implementation of control of distributed parameter systems. Since the field has seen an explosive growth over the last decades (and it continues to grow), perhaps this is an appropriate time to take an account of the various developments in an unified manner, including some possible dimensions of future research.

Surveys of theoretical, as well as application, papers on DPS control can be found in Ray (1978), Balas (1983), Curtain and Zwart (1995) and Lasiecka (1995). In addition to these available literature (which are fairly old), this paper presents the development of control strategies for distributed parameter systems (DPSs) until very recently. It also attempts to present the developments in a chronological order. The survey work has been organized as follows: first we briefly present some of the early developments, where we restrict ourselves up to 1989. The developments 1990 onwards have been introduced as recent developments and discussed more elaborately. This demarcation between early and recent developments is non-technical and it is based on the popularity of DPS. Various conferences have been dedicated to the control of DPS towards the end of twentieth century, which increased the number of readers and researchers in the field of DPS. At the end some open research topics and possible future directions have been discussed before including the concluding remarks.

2. Early developments (1960–1989)

Literatures pertaining to the control of DPS can be found as early as 1960. Butkovskiy and Lerner probably published the first paper in this field in 1960 (Butkovskiy & Lerner, 1960), deriving a general maximum principle for a class of distributed parameter systems. This was followed by a series of papers from Butkovskiy (1961a, 1961b, 1966). These works were concentrated mainly on problem formulation and the maximum principle for a distributed parameter describable by a set of integral equations. All of these developments can be found at one place in a book written by Butkovskiy, the English translation of which was published in 1969 (Butkovskiy, 1969).

In 1964 Wang and Tung published their pioneering work (Wang & Tung, 1964), which laid the formulation for further development

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towards a unified control theory for DPS. In close parallel to the development of lumped parameter system theory, the paper attempted to give precise mathematical description of DPS in term of a set of partial differential equations. The authors discussed the concepts of controllability and observability. Besides, they formulated the optimal control problem and derived the optimality conditions for a wide class of DPS. The paper also discussed some numerical solution tools with discretized approximations. Later in 1966, in a separate paper Wang addressed the problem of stability of DPS with feedback controllers directly in the framework of PDEs, using Lyapunov techniques without resorting to approximations (Wang, 1966). These developments lead to a series of papers by various authors proposing many solution techniques.

The paper by Sakawa in 1966 (Sakawa, 1966) discusses two methods for the optimal control of heat equation. One of the two is the application of variational method, whereby Fredholm's integral equation of first kind is derived as a necessary condition for optimal control. The other method reduces the problem suitable for application of linear and nonlinear programming for the synthesis of optimal control. In 1966, Axelband (Axelband, 1966) analyzed the tracking problem of a general model for a linear DPS, with bounded control input. The optimal solution was shown to exist and a control synthesis tool was presented by parameterization, using convex programming. In the same year, Sage and Chaudhuri (1967) proposed to use gradient and quasi-linearization computational techniques for the synthesis of optimal control, using the necessary conditions of optimality proposed by Wang (Wang & Tung, 1964). Based on various discretization schemes, the paper addressed (perhaps for the first time) computational tools for a class of nonlinear DPS. For the regulation problem of linear systems, the paper also proposed to incorporate the techniques already developed for the lumped parameter systems by suggesting only spatial discretization.

In 1967, Kim and Erzberger proposed a Hamilton–Jacobi approach to the one-dimensional wave problem, with quadratic cost function (Kim & Erzberger, 1967), having boundary control. The authors came up with a set of Riccati equations whose solution gives the closed loop optimal control. The authors also presented a solution technique for the Riccati equations based on the eigenfunction representation of the Green's function, which leads to the necessity of solving only a set of ordinary differential equations. Following the idea of Kim and Erzberger, in 1969 Alvarado and Mukundan (Alvarado & Mukundan, 1969) obtained a Riccati-type matrix partial differential equation for the problem of a furnace, heating a one-dimensional slab, and presented an approximation technique to solve the equation.

In 1970 Graham (Graham, 1970) presented a relatively simple formulation of the one-dimensional heat conduction problem using a sample-data model of the problem. This permitted the solution of Hamilton–Jacobi formulation by two approaches, namely the matrix Riccati method and the Kalman's equation method. Another paper from Goldwyn, Sriram and Graham appeared in 1970 (Graham, 1970), where the authors showed the applicability of Laplace transform for the determination of time optimal control for hyperbolic class of problems. This paper demonstrated the effect the nature of a PDE has on the form of optimal control. A suboptimal synthesis approach was presented in the paper, which did not always lead to bang-bang form of control. Hassan and Solberg published a paper in 1970 (Hassan & Solberg, 1970) that discussed the optimum control problem of linear DPS with quadratic cost function in a discrete time framework. The authors, using orthogonal series expressions for a Riccati-type functional equation, proposed a scheme of recursive functional expressions involving Green's function matrices. In 1970 itself Julio published a paper (Julio, 1970b) presenting a different technique to compute the optimal control for linear DPS, which avoided the need to solve the PDE. In another paper in the same year (Julio, 1970a), the author addressed the problem of convergence of the discretized approximate solutions to the continuous solution as the discretization interval goes to zero.

In 1972 Chaudhuri published a paper (Chaudhuri, 1972), which discussed the optimal control of DPS having wave-type phenomenon (hyperbolic systems). The paper did not propose any new computational tool for optimal control synthesis and rather extended the computational tools published earlier by the author in Sage and Chaudhuri (1967) for nonlinear hyperbolic systems. However it discussed many associated issues, such as the mathematical description of the propagation and growth of the wave generated and the statement of maximum principle of the system generating waves. It also addressed a few problems associated with discretization schemes for such systems.

A successive approximation algorithm was presented by Zone and Chang in 1972 (Zone & Chang, 1972), for a general class of nonlinear DPS with nonlinear functional boundary conditions, based on the second-order expansion of the performance index. Necessary and sufficient conditions for the convergence of the algorithm, which are independent of the system dynamics, were also shown to be satisfied. The brief paper from Davis and Perkins (1972) is devoted to the optimal control design of a class of DPS with separable controllers. A separable control has fixed functional dependence on the spatial variables and free functional dependence on the time variable. The authors derived the optimal conditions for such systems as a special case of the results due to Wang and Tung (1964) and solved a plasma containment regulator problem.

The problem of observability and optimal sensor location for linear DPS is addressed by Yu and Seinfeld in 1973 (Yu & Seinfeld, 1973). Curtain and Pritchard published two papers in 1974 (Curtain & Pritchard, 1974, 1975). In these papers, the authors developed the Riccati operator following the infinite-dimensional operator theory. This provided a basis to synthesize the optimal control for linear DPS directly in the infinite-dimensional framework. For a comprehensive treatment of this concept of infinite-dimensional operator theory based optimal control for linear systems one can refer to the book written by Curtain and Zwart published in 1995 (Curtain & Zwart, 1995).

Balas developed a feedback control scheme for the class of systems given by the generalized wave equation in 1978 (Balas, 1978). The problem addressed considered a number of point force actuators and point sensors. The feedback controller was developed for a finite number of models of the flexible system. The paper also presented the controllability and observability conditions necessary and examined the control and observation spillovers due to residuals. Some remedies for the spillover problem were also suggested (including a phase-locked loop pre-filter).

Pritchard and Zabczyk addressed stability and stabilizability problems of infinite-dimensional system in 1981 (Pritchard & Zabczyk, 1981). Meanwhile, in 1979 Gibson wrote a paper (Gibson, 1979), which presented the Riccati integral equations for optimal control problems on Hilbert spaces. In 1981 the author wrote another paper (Gibson, 1981), which can be thought of as a general framework for convergence in the context of regulator problems. Based on the ideas presented in Gibson's papers, Banks and Kunisch wrote a good technical note in 1984 (Banks & Kunisch, 1984) presenting an approximation framework for the computation of Riccati operators, which was shown to converge to the Riccati operator in feedback control for parabolic type linear infinite-dimensional systems.

In 1988 the Ph.D. dissertation of Pourki addressed the problem of stability, using semigroup theory (Pourki, 1988). The dissertation presents a Lyapunov stability analysis technique for DPS. The examples considered include parabolic and hyperbolic classes, indicating the suitability of the method for both classes of DPS. The thesis also considers the controllability and observability issues and comes up with a formulation for finite order feedback control.

In 1989 Futagami et al. edited a volume containing a collection of papers on modeling and simulation of DPS (Futagami, Tzafestas, & Sunahara, 1989). This includes a paper from Tassan (1989) on "same basic multigrid algorithms for DPS", which serves as a general strategy for numerically solving the continuous problems. The algorithms basically involve iterative loops between coarse and fine grids that lead the solution to the level of discretization errors. The volume also includes a paper from Shimemura, Uchida, and Kubo (1989), in which the authors proposed a designed method for a linear quadratic regulator for DPS that locates the poles in a specified region. A feedback control law was proposed to be constructed from a finite-dimensional Riccati equation. The method presented a tool to calculate the closed loop poles apriori, so that a regulator can be designed having desired degree of exponential stability. The volume includes a paper from Yoshida (1989) that presents a method for computation of finite-dimensional controllers, taking into account the spillover effects due to such finite-dimensional approximations. In this method, the spillover is formulated as a control structure constraint and the control is synthesized by following a minimum norm suboptimization technique. In 1989 itself, another control design method was proposed by Matsumoto and Yoshida (1989) for parabolic type DPS. This was done by approximating the system as a lumped parameter by using integral transforms.

3. Recent developments (1990 onwards)

In 1990 Bernstein and Rosen (Bernstein & Rosen, 1990) proposed a new finite-dimensional approximation scheme for the optimal fixed-order compensation of DPS, which used the so-called Bernstein/Hyland optimal projection theory (Hyland & Bernstein, 1984) in Hilbert space. The methodology yields fixed-finite-order controllers. This approach involves constructing a sequence of approximating finite-dimensional subspaces of the original infinite-dimensional Hilbert space, along with corresponding sequences of bounded linear operators that approximate the given input, output and system operators. Choosing bases for the subspaces and applying the optimal projection theory, a sequence of matrix equations is obtained. Then the authors propose to use available numerical techniques (e.g. Homotopic continuation algorithm Richer, 1987), to compute the sequence of approximating gains.

In 1991 Li and Fadali published a good brief paper (Li & Fadali, 1991), leading to the development of two-dimensional optimal control theory that parallels the Hamiltonian formulation of one-dimensional optimal control. The authors gave a theorem stating the state equation, costate equation, stationary equation and boundary conditions for such system. Though the development was in discrete domain, and hence not exactly addressing the PDE-driven problems in strict mathematical sense, still it can be thought of as a tool for the optimal control design, starting with an appropriate discretized form of the system dynamics and cost function.

In 1992 Helmiki et al. established various connections between linear time-invariant DPS in continuous time and the zero-order hold discrete time equations (Helmicki, Jacobson, & Nett, 1992). This was done both in time and frequency domains. In 1993 a theoretical paper by Fattorini (1993) presented the maximum principle of a class of semi-linear parabolic DPS in Banach spaces. The results apply to system described by nonlinear heat equation and reaction–diffusion equation in L^1 and L^∞ spaces. In 1994 Burns and King (Burns & King, 1994) addressed the optimal sensor location and estimator design problems leading to a robust control design of DPS.

In 1995 Lasiecka wrote a paper (Lasiecka, 1995) citing a historical perspective of the deterministic control of PDE-driven systems. Though it was a short paper, it provided a unified overview of the literature on this widely scattered and mathematically nonhomogeneous field by that time. However, the detailed survey was towards the boundary and point control problems. In 1995 itself Alli and Singh (1995) derived the closed form solutions for the class of DPS governed by the linear wave equation. A frequency domain approach (using Laplace transforms) was used to arrive at a bang-off-bang type optimal solution.

Joshi et al. wrote a paper in 1995 (Joshi, Speyer, & Kim, 1995) and addressed the stabilization problem of plane Poiseuille flow. The authors first constructed a finite-dimensional model following the Galerkin projection approach using Fourier functions as basis functions. Then they applied the Linear Quadratic Gaussian (LQG) robust control design technique. The authors also proposed a reduced-order LQG control design that stabilizes the system to a prescribed degree.

Meanwhile Choe and Chang published a paper in 1995 (Choe & Chang, 1995) comparing the two methods of optimal control synthesis, namely the PDE approach and Integral Equation (IE) approach, for the problem of a tubular reactor with axial dispersion. The authors found that for the problem under consideration the approach with PDE description was preferable, even though the IE approach provided the exact solution where as the PDE approach provided only an approximate solution. This was because, the authors observed, one of the time-consuming steps in IE approach is the determination of adequate number of positive roots of an accompanied characteristic equation, with sufficient accuracy. On the other hand numerical programming of PDEs was found to be easier and more flexible. Later in 1998 the same authors wrote another paper (Choe & Chang, 1998), which attempted to merge the PDE and IE approaches of optimal control synthesis for the same tubular reactor problem. This method was found to be more efficient than either of the methods, both in computing procedure as well as computing time.

In 1996 Farahi, Rubio, and Wilson (1996) considered the existence of boundary control for linear wave equation. After modifying the problem into a problem consisting of the minimization of a linear functional, the problem was converted to a finite-dimensional linear programming problem. The solution to this linear programming problem was then used to construct a piece-wise constant control.

By 1997 the Conference on Decision and Control (CDC) as well as the American Control Conference (ACC) started devoting special sessions on DPS. To mention a couple of papers from the proceedings of CDC-1997, the paper by Banaszuk (Banaszuk, Hauksson, & Mezic, 1997) proves the existence and uniqueness of solution in the Sobolev space for the so-called Moore-Greitzer nonlinear PDE, with some "mild conditions", after reformulating the problem in the Banach space. The paper also presents a design methodology based on back-stepping idea, using the modified formulation. The paper by Drakunov and Barbieri (1997) presents a stability analysis for DPS described by a class of multi-dimensional PDEs. This paper also presents a control design approach using model expansion. The approach first finds a manifold in the system's infinite dimensional state space such that if the system is confined to this manifold, it has the desired properties. Then the control makes that manifold an area of attraction for the closed loop system. Based on the idea of sliding mode control, this methodology steers the states towards the manifold and once reached, maintains them within it. The paper from Bameih (1997) considers a special class of spatially invariant DPS and presents a variety of optimality criterion. The paper also shows that the optimal controllers retain the same spatial invariant characteristics. Meanwhile in 1997 Banks, Smith, Brown, Silcox, and Metcalf

(1997) addressed some of the issues regarding the experimented implementation of PDE-based controllers. The paper though did not contain any new theoretical results, it attempted to answer a few doubts about the implementability of some of the existing methods.

In a series of papers in CDC-97 and ACC-98 (Christofides, 1997; Christofides & Armaou, 1998; Christofides & Baker, 1998; Christofides & Daoutidis, 1997), Christofides and the co-authors addressed several class of DPS problems. In reference Christofides and Daoutidis (1997) the authors proposed a control design tool for a class of quasi-linear parabolic PDE-driven systems, for which the eigenspectrum of the spatial differential operator can be partitioned into a finite dimensional slow spectrum and an infinite-dimensional fast one that are stable. The methodology uses Galerkin approach to come up with a suitable ODE system for the PDE dynamics and design a nonlinear output feedback controller that guarantees stability while making the system follows a desired response. The paper (Christofides, 1997) proposes a robust control design technique for the same class of DPS. The paper (Christofides & Baker, 1998) is an improvisation of Christofides (1997), whereby the authors present a robust control design technique based on output feedback. In the paper (Christofides & Armaou, 1998), the authors design a finite-dimensional nonlinear controller for a two-dimensional nonlinear Navier–Stokes equation. Later in 2001, Armaou and Christofides published a paper (Armaou & Christofides, 2001) presenting a robust nonlinear static output feedback control design tool for DPS governed by quasi-linear parabolic PDE systems. Many of the innovative ideas from Christofides et al. are summarized in Christofides (2001).

The paper by D'Andrea in ACC-98 (Andrea, 1998) uses linear matrix inequality theory and comes up with a decentralized control strategy for multi-dimensional but linear DPS. In 1998 itself Balas presented a paper (Balas, 1998), which uses the theory of semigroups in the infinite-dimensional state space to address the stability problem of the actual DPS in closed loop and presents stability bounds in both time and frequency domains. However the author concludes that the frequency domain conditions can more easily be tested in practice. In 1998 two experimental papers also came out in the literature. The paper by Yoshida and Matsumoto (1998) addresses a chemical reactor problem. The paper by Yang and Jeng (1998) address a structural control problem. The short paper by Sadek and Bokhari in 1998 (Sadek & Bokhari, 1998) presents a different way of optimal control synthesis using orthogonal polynomials. In this approach the author used the standard model space technique to form an analogous lumped-parameter optimal control problem. Then they proposed to use finite interpolating orthogonal polynomials to present a simplified computational method for evaluating the optimal control. The paper, however, addresses only self-adjoint linear parabolic class of DPS. Writing a separate paper in 1995 (Sadek & Yurekli, 1995) Sadek and Yurekli addressed the general problem of combining the open-loop and closed-loop controls for a system of multi-dimensional PDEs describing large structural systems. This approach first designs a set of open loop controls. Then the feedback parameter describing the closed-loop control is computed by minimizing energy functional over all possible feedback parameters.

In 1999 Godasi, Karakas, and Palazoglu (1999) studied the control nonlinear DPS in the framework of symmetry groups and group invariant solutions of a differential system. Based on this idea, Palazoglu and Karakas published a brief paper in 2000 (Palazoglu & Karakas, 2000). They proposed both continuous and discontinuous controller design. Symmetry group has been used to determine the group-invariant solutions of a differential system to facilitate the formulation of control strategies. The invariance of the solution space leads to the stability condition that produces the control law. Essence of their study was to present a generalized quantitative expression of the stability condition for any nonlinear system. Further, an invariance condition in the so-called "prolonged space" of the differential system provided a framework for the distributed control law.

In 1999, Matsuno and Murata (1999) developed a direct output feedback control law based on proportional, derivative and strain feedback (PDS) for a one link and a two link flexible arm based on the distributed parameter model. They introduced Lyapunov function related to the total energy of the distributed parameter system and derived a simple sensor output feedback control law without approximated finite-dimensional model at the controller design phase. Stability and convergence proof of the closed loop distributed parameter system were shown using the invariance principle and the characteristic of the differential operator. In order to demonstrate the validity of the derived model and the effectiveness of the proposed control law experimental results have been reported.

A frequency domain input–output approach has been proposed by Reinschke and Smith in 1999 to linear time invariant distributed parameter systems (Reinschke & Smith, 1999). The approach was a generalization of H_∞ control in the sense that the second norm was used for both the space and time-dependence of signals, which was used to take account of the spatially distributed nature of a system's input and output signals. Finite-dimensional approximation of the LTI systems with distributed nature has been considered for controller design. The authors proposed two controller synthesis methods similar to the standard H_∞-synthesis techniques of "H_∞ loop-shaping" and "μ-synthesis" on the basis of finite-dimensional distributed linear time invariant systems.

In 2000 Alli and Singh wrote a paper (Alli & Singh, 2000) on the feedback control of wave equation. This paper addressed the problem of designing collocated (having same controller and actuator locations) and non-collocated controllers. The authors considered the control design problem for a uniform bar without structural damping, which made the closed loop system stable. A Lyapunov based approach was adopted for the collocated problem, where as a frequency domain approach was used for the non-collocated case. The paper also proposed some stability conditions for both cases. In the same year Khalid (2000) extended his spreading control law developed for linear systems and reported earlier in Jai and Kassara (1994, 1997) to semilinear distributed parameter systems. The authors this time used the spreading control as a selection of the feedback map. In the paper, a minimum energy spreading control law was derived for systems with affine dependence upon the control by using a parametrized constrained optimization technique along with some facts of set-valued analysis.

In 2000 itself Byrnes, Lauko, Gilliam, and Shubov (2000) extended the geometric theory of output regulation, introduced in Isidori and Byrnes (1990) and Francis (1977) for solving the state and output feedback regulator problems for infinite-dimensional linear control systems, assuming bounded control and observation operators. The application of the model was restricted to the case when the reference signal and disturbances were generated by a finite-dimensional exogenous system (for details on exogenous systems one can refer to Francis, 1977). In particular, the paper showed that the full state feedback and error feedback regulator problems can be solved, under the standard assumptions of stabilizability and detectability, if and only if a pair of regulator equations is solvable. The regulator equations form a system of Sylvester-type operator equations subject to extra side constraints. For the examples taken, the regulator equations reduced to a system of linear ordinary differential equations, which, in general, were solved numerically off-line to obtain approximate feedback control that work very well in practice. Later in 2003, Byrnes, Gilliam, Isidori, and Shubov (2003a) extended the technique to a set point boundary control of nonlinear parabolic DPS. This paper defined a problem of output regulation for nonlinear distributed parameter systems, in the non-equilibrium settings. It also gave necessary and sufficient conditions for a controller to solve the output regulation problem. Basically the paper extended the non-equilibria theory of nonlinear output regulation, so as to apply to the set-point control of certain nonlinear distributed parameter systems. The proposed method has been applied on a heat equation and for a class of nonlinear reaction–diffusion equations. The method still remains promising but as discussed by Meurer and Zeitz in 2004 (Meurer & Zeitz, 2004), the main restriction was given by the fact that the reference trajectory should be generated by a neutrally stable finite-dimensional exogenous system (the external system that generates tracking signal for a fixed plant so that the output tracks the reference signal), which is not practical for the realization of transitions between stationary profiles or non-periodic reference trajectories. For an example, DPS control problems arising in biological and chemical engineering are usually highly nonlinear and require various set-point changes during startup, operation and shutdown.

In 2001 Fard and Sagatun published a paper (Fard & Sagatun, 2001) that came up with an exponentially stabilizing nonlinear boundary control law to stabilize a nonlinear beam transverse vibration. The advantage was the control law used measurements from the boundary only. This control design took into account the coupling between longitudinal and transversal vibrations. Further, the authors proved the stability of the closed loop system taking the help of Lyapunov's direct method.

A variety of neural network based optimal control synthesis approach for DPS systems has been proposed by Padhi and Balakrishnan in a series of papers followed 2000 onward. The method has been initialized in the 2000 American Control Conference (Padhi & Balakrishnan, 2000). An important contribution of the study was the derivation of the necessary conditions of optimality for distributed parameter systems, described in discrete domain, following the principle of approximate dynamic programming. Then the derived necessary conditions of optimality were used to synthesize infinite time optimal neuro-controllers in the framework of adaptive-critic design. In 2001 (Padhi, Balakrishnan, & Randolph, 2001) the authors extended the work and published a similar paper in 'Automatica', which included a comparison with linear quadratic regulator problem for the diffusion equation that has a Riccati-operator based solution. For the synthesis of the controller, the method proposed two sets of neural networks: the set of action networks captures the mapping between the state and control, while the set of critic networks captures the mapping between the state and costate. Note that the construction of the networks and the synthesis of the controller were relatively free of simplified assumptions (like linearized models).

Probably this survey would remain incomplete without mentioning a powerful model reduction technique, known as Proper Orthogonal Decomposition (POD) followed by its usage in Galerkin Projection and control design. The method was first proposed by Karhunen (1946) and by Loeve (1945) independently. For this reason this technique is also called as Karhunen–Loeve (K–L) expansion. However the technique gained significance much later. Lumley used it in the name of POD, to study turbulent flows in 1967 (Lumley, 1967). The method gained further acceptability when technique of "snapshots" got incorporated into it by Sirovich (1987). The power of the technique lies in the fact that it creates "problem oriented" basis function, which when used in conjunction with a reduced basis method (for example, method of Galerkin's projection) leads to a very low dimensional representation of the DPS with high accuracy. A number of application papers based on this technique continue to appear in the current literature. A few of them include the paper by Burns and King (1998) on the development of a feedback control law of hybrid DPS, the NASA report by Ravindran (1999) on the optimal control of fluids, the paper by Banks et al. (2000) on the feedback control of a thin shell model and the paper by Singh, Myatt, and Addington (2001) on the development of an adaptive feedback linearizing control for a two-dimensional nonlinear flow.

In 2002 Shang, Forbes, and Guay (2002) proposed a Model Predictive Control method for DPS governed by hyperbolic PDEs, using the method of characteristics. It showed that the proposed MPC possesses the advantage of high computation efficiency and desired performance because of the method of characteristics. 2002 also show the wavelet based point-wise PDE control strategy proposed by Gao, Gu, and Zeng (2002). Haar wavelets have been used for orthogonal function approximation of distributed parameter systems to deal with the optimal point-wise control. The differential operational matrix and the product integrated operational matrix of Haar wavelet base have been proposed. Similar approach of converting a DPS to lumped parameter system using orthogonal Haar Wavelets, has been reported later in (Ding & Gu, 2004) and (Ding & Gu, 2006) for predictive control of DPS systems.

In 2002, Zheng, Hoo, and Piovoso (2002) reported the use of SVD-KL that combines singular value decomposition (SVD) theory with Karhunen–Loeve (K–L) expansion technique to develop low-order models of nonlinear, one-dimensional PDEs, when there is no available exact model of the system. The SVD-KL were used to capture intermediate and low frequency modes important for controller synthesis. The input–output model that came from the application of the SVD-KL method was used to design the dynamic matrix controller and PI controller. It was shown that satisfactory closed-loop performance of the nonlinear DPS could be obtained using a DMC designed using the finite order model of the system. In the same year, Fung, Wu, and Lu (2002) proposed an adaptive boundary control to an axially moving string system using a mass-damper-spring controller at its right-hand side boundary. Unknown parameters appearing in the system equation were assumed constant and estimated on-line by using adaptation laws. Stability were guaranteed in Lyapunov sense. Finally, the performance of the proposed controller was demonstrated by numerical simulations where finite difference technique has been used for PDE approximation.

In 2003, King and Hovakimyan (2003) presented a paper where finite-dimensional Linear Quadratic Gaussian (LQG) controller has been augmented for a distributed parameter system, with an adaptive output feedback element. The adaptive parameters were determined in neural network framework. The theory was discussed for a problem concerning control of vibrations in a nonlinear structure with bounded disturbances. In another instance, Toshkova and Petrov (2003) discussed an algorithm for linear quadratic optimal control synthesis for parabolic distributed parameter systems. The distributed parameter system was reduced to a lumped parameter system by applying the generalized finite integral transformation technique. The control law was related to obtaining solutions of algebraic Riccati equation, which was realized by using neural networks in real time. Another neural based application can be found in Prokhorov (2003) where the authors proposed to use the framework of backpropagation through time (BPTT) to create optimal feedback neuro-controllers for DPS. The neuro-controllers obtained for discretized DPS in the infinite-horizon regulator setting were applicable to a broad set of initial states (an envelope of initial state profiles). They compare the technique and results with results reported by Padhi and Balakrishnan in (Padhi & Balakrishnan, 2000), and (Padhi et al., 2001). Discretization of DPS equations before utilizing the back-propagation principle to obtain BPTT equations was performed in the method and then BPTT equations were used directly to compute derivatives for subsequent training of a set of neural networks.

In 2004, Demetriou and Fahroo (2004) discussed an adaptive control mechanism for a class of second-order distributed parameter systems that can be stabilized via static output feedback. The adaptive estimates of the unknown feedback gains were utilized for the convergence of state position and velocity in Lyapunov sense. By further assuming that structured perturbations of the damping and stiffness operators, often describing modeling uncertainties, were always present in the system, a modification to the proposed adaptive control law was included in order to address these un-modeled uncertainties. In the same year, a coupled Green-Galerkin numerical method proposed by Alaeddine and Doumanidis (2004) for infinite-dimensional thermal conduction systems. The method used Galerkin optimization of an energy index employing spatial and temporal convolution of distributed Green's fields. In the analysis presented, the developed Green-Galerkin method was applied to a one-dimensional thermal observation problem. The article investigated the development of the method and was able to address the fundamental questions of thermal control and observation in infinite-dimensional thermal conduction systems.

A two degree-of-freedom feedforward and feedback design approach was proposed by Meurer and Zeitz (2004) in 2004, for boundary control of tubular bio-reactor model with nonlinear reaction rate. The method used an advanced feedforward control, in contrast to geometric approach used by Byrnes et al. (2000), Byrnes, Gillam, and Shubov (2003b) in 2000, to achieve the tracking performance of a control loop complemented by a feedback control, to compensate for model errors and exogenous disturbances. For the design of the feedforward part, formal power series were used in conjunction with sophisticated summability methods. This allows an extension of the formal power series approach to nonlinear and in particular convection dominated DPS, the latter playing an important role in chemical engineering applications. Since the tracking behavior was mainly determined by the feedforward control, the output feedback control has been reported to be sufficient to account for offsets and disturbances or stabilization along the desired output trajectory.

An optimal dynamic inversion strategy for DPS has been proposed by Padhi and Balakrishnan (2007) in 2007. The combination of dynamic inversion principle and optimization theory has been used to design a stable controller for a class of one-dimensional nonlinear distributed parameter systems incorporating an optimal control allocation strategy. This approach does not demand any approximation either of the system dynamics or of the resulting controller. Furthermore, the method provided a closed form solution for the controller feedback and therefore suitable for practical applications.

In 2006 a four paper special issue on 'Advances in Robust and Nonlinear Control of Distributed Parameter Systems' has been published in 'International Journal of Robust and Nonlinear Control'. In the first paper, Strub and Bayenz (2006) proved the existence and uniqueness of a weak solution to the Lighthill-Whitham-Richards partial differential equation (LWR PDE) in the presence of boundary conditions. A weak formulation of the boundary conditions makes the problem to be well-posed. The existence of the solution results from the convergence of the Godunov scheme. A highway traffic flow scenario presented in the paper illustrates the applicability of the method. Boundary control of the LWR PDE was applied to a highway optimization problem with data from highway Interstate-80 (in USA) obtained from the Berkeley Highway Laboratory. Note that the boundary control was used to minimize travel time on a given stretch of the highway.

The second paper, written by Dubljevic, El-Farra, Mhaskar, and Christofides (2006), presents and compares a number of model predictive control formulations for control of linear parabolic PDEs with state and input constraints. Modal decomposition techniques are used to derive finite-dimensional systems that capture the dominant dynamics of the PDE. A number of model predictive control (MPC) formulations, designed on the basis of different finite-dimensional approximations, were then presented and compared. The closed-loop stability properties of the infinite-dimensional system under the low order MPC controllers are analyzed and sufficient conditions that guarantee stabilization (satisfying state constraints) are shown.

In the third paper, Demetriou and Fahroo (2006) considered adaptive control of a class of structurally perturbed second-order distributed parameter systems in which both partial position and partial velocity measurements were assumed available. The adaptive controller for the second-order system utilized both online estimates of the static feedback gains and online approximators of the unknown structured perturbations. The perturbations were canceled via feedback, and the adaptive controller directed the closed-loop dynamics match those of the reference model. The control objective was to design an adaptive controller so that the plant state followed the state of a second-order reference model despite the presence of the perturbation terms.

The fourth paper, written by Krstic (2006), is of tutorial nature. The paper presents a catalogue of approaches for the design of adaptive controllers for PDEs controlled from the boundary (boundary control) and containing unknown destabilizing parameters affecting the interior of the domain. The paper differentiates between two major classes of schemes: Lyapunov schemes (published in Smyshlyaev & Krstic, 2006) and certainty equivalence schemes. Within the certainty equivalence class two types of identifier designs were pursued: passivity-based and swapping designs (published in Smyshlyaev & Krstic, 2007a; Smyshlyaev & Krstic, 2007b).

Immonen (2006) designed a controller for linear distributed parameter systems (with bounded control, observation and feedthrough operators) in 2006, which under certain assumptions, achieved asymptotic tracking of arbitrary bounded uniformly continuous reference signals in the presence of disturbances. An independent feedforward-feedback controller has been reported where the dynamic feedback part was used to stabilize the closed-loop system consisting of the plant and the controller, whereas the feedforward part was tuned using the regulator equations to achieve the regulation of desired signals.

Among others works in 2006, we refer to application of fuzzy logic based control by Li, Zhang, and Li (2006, 2007) for catalytic packed-bed reactor (infinite dimensional system). The papers use 3D fuzzy membership function to take care of the spatial variability of the DPS. Different to the traditional fuzzy logic controller, the authors use multiple sensors to provide 3D fuzzy inputs and proposes the inference mechanism with 3D nature that can fuse these inputs into a so-called "spatial membership function". Application of boundary control technique coupled with predictive control by Ding and Gu (2006) for distributed parameter system control. Here, orthogonal Haar wavelets based discretization of DPS has been done to transform boundary predictive control algorithm of time-discrete first-order linear distributed parameter system into boundary predictive control issue of lumped parameter system. The problem of constructing model reference adaptive H_∞ control for distributed parameter system of hyperbolic type was considered in the paper by Miyasato (2006), where the proposed control scheme was constructed from finite dimensional controllers. The stabilizing control signal was added to regulate the effect of spill-over terms, and it was derived as a solution of certain H_∞ control problem where spill-over were considered as external disturbances to the process.

In 2006, Kim and Bentsman (2006) presented a multi-resolution based technique for the finite-dimensionalization of the controller parameter adaptation laws in adaptive control of DPS. This technique permits efficient incorporation of the prior knowledge of the specific plant parameter characteristics (such as non-smoothness) into controller implementation through the choice of parameter approximation basis, yielding a high fidelity parameter representation by a small number of basis coefficients. For this purpose, a new tool the "multiresolution Lyapunov functional" has been introduced. Using this the existence of the wavelet-based finite-dimensional parameter adaptation law is proposed, which provides the desired tracking accuracy while retaining the well-posedness of the closed-loop system with the infinite-dimensional plant. The benefits of the technique in both real-time and off-line performance enhancement of the control law, such as reduction of computational demand and increase in the output convergence rate unaccompanied by the corresponding increase in the control effort are demonstrated.

In 2006 Nguyen and Egeland considered the output feedback stabilization problem for a class of second-order distributed parameter systems without distributed damping (or non-strictly positive distributed damping). Exponentially stable observer and controller were designed. The main analysis tool was semi-group theory (Nguyen & Egeland, 2006).

In a separate instance, Borggaard (2006) studied the development of reduced-order models for nonlinear distributed parameter systems in 2006 itself. The method was based on Galerkin projection, but the reduced-basis vectors were optimal for the dynamic model, found by minimizing the error between given full-order simulation data and the reduced order model. This was achieved by formulating the basis selection problem as an optimal control problem with the reduced order model as a constraint. This methodology allows a natural extension of reduced-order modeling ideas to nonlinear systems.

In 2007, the paper by Garcia-Sanz, Huarte, and Asenjo (2007), introduced a new simple quantitative robust control design technique applicable to one-point feedback controllers for DPS with uncertainty. The paper proposes a new set of transfer functions (TFs) that describe the relationships between the inputs and outputs of the system. These points were chosen to be spatially distributed at the relevant points where the inputs and the outputs of the control system were applied (actuators, sensors, disturbances and control objectives). Based on these TFs, the paper extended the classical robust stability and performance specifications to the DPS case and presented a new set of quadratic inequalities to define the quantitative feedback theory bounds.

A general procedure for parabolic PDEs with spatially continuous backstepping based boundary control, introduced in Smyshlyaev and Krstic (2004) by Smyshlyaev and Krstic has been proposed by Krstic et al. in 2007 (Krstic & Smyshlyaev, 2007). The paper contained a discussion of the main design. The paper concluded with the application of the backstepping method to the Schrodinger equation and first-order hyperbolic PDEs (the transport equation and its derivatives).

In 2007 Smyshlyaev and Krstic proposed an adaptive version of the backstepping based boundary control through a couple of papers (Smyshlyaev & Krstic, 2007a, 2007b). An output feedback adaptive control scheme for two benchmark parabolic PDEs motivated by a model of thermal instability in solid propellant rockets has been proposed in the papers. Both benchmark plants were unstable and were controlled from the boundary. One plant has an unknown parameter in the PDE and the other in the boundary condition. The paper (Smyshlyaev & Krstic, 2007a) introduces the novel approach to adaptive control of PDEs where a parametrized family of boundary controllers can be combined with "swapping gradient" identifiers to yield global stability of the resulting nonlinear PDE system. Only the state-feedback problem was considered there. For a different, narrower, class of systems, the output-feedback problem can be solved by this method, which is illustrated on two benchmark examples in the short paper (Smyshlyaev & Krstic, 2007b).

4. Open problems and possible future directions

The research on control of DPS has experienced a phenomenal growth and various innovative methods have been developed over the last few decades. However, the area remains a fertile field for research. There is immense potential in this field of research and an extensive list of all possible future directions is perhaps an impossible task. In this section, however, we attempt to include a brief discussion about some of the problems that in our view are potential future directions of research.

- Many engineering solution techniques presented in various literature rely heavily on 'discretized solution' of the original problem. Even though these solutions are mostly reliable, precise conditions about the step sizes used in the process of discretization needs some special attention. Conditions on the step sizes for the linear DPS are typically available, whereas for the nonlinear systems such conditions are not available. It is an important dimension of research since too big a step size invokes accuracy issues whereas small a step size invites implementation concerns. A precise idea of tolerable step sizes in general would be potential dimension of research.
- One of the important issues in synthesizing discrete optimal controllers for distributed parameter systems is the question of accuracy of the discretized problem itself. Even if the solution to the discrete problem exists, to address the question of accuracy one has to answer whether the solution to the analogous discrete problem will eventually converge to the continuous problem in limiting sense, as the discretization intervals tend to zero. This question has been addressed in (Julio, 1970a) to come up with sufficient conditions for linear DPS. However to the best knowledge of the authors, no precise results exist for this problem for general nonlinear DPS.
- Even though many numerical methods have been proposed, there is no systematic approach towards development of closed form solution of control in general for optimal control design for nonlinear problems. Development of such closed form solutions (which can possibly rely on infinite-dimensional operator theory) will have a great impact.
- Efficient computational techniques for reduced-order modeling need to be developed further from the perspective of control design. Even though the proper orthogonal decomposition (POD) based problem-oriented basis function design has emerged as a popular tool for model reduction, questions like the validity of the snapshots (using which the basis functions are designed) in closed loop is a major concern since the solution behavior of s dynamic system can be completely different as compared to the open loop solution. Even though some ideas like Re-POD, adaptive POD, etc. have appeared in the literature, in our opinion this issue needs further attention to efficiently incorporate the closed loop solution behaviour in the basis function design.
- Another important issue in connection to reduced order modeling and its real time application is the effect the unmodeled dynamics. Spillover effects that arise due to non observing and not controlling the unmodeled dynamics need better understanding and clarification.

- Another area that needs further research is the issue of stability and stabilizability. Many finite-dimensional stability theories in the sense of Lyapunov have been applied to infinite dimensional systems as well. However, it is well-known that many important classes of mechanical systems cannot be stabilized in the sense of Lyapunov, while the concept of partial stability naturally appears (Luo, Guo, & Morgul, 1999; Vorotnikov, 1998). Characterization of the partial stability and stabilizability of nonlinear infinite-dimensional system are still less developed and need further attention.
- Intelligent control of DPS using neural networks, fuzzy logic, etc. are computationally less expensive, relatively easier to implement and comparatively less sensitive to noise. However, there are many issues in such designs (like selection of basis function, appropriate choice of fuzzification, etc.), which need further attention.
- Solution techniques emerging in various fields like computational fluid dynamics, finite element method (FEM), element free techniques, etc. need further attentions because of challenging requirements like irregular boundaries, non-homogeneous boundary conditions, requirement of grid-free solutions, etc.
- Another important area that needs to be explored is perhaps the field of 'impulse control', where the control action enters to the system only at intermittent instants of time (and possibly at various spatial locations as well). This area, which holds great promise in biomedical applications, is at its infancy and definitely opens up a great opportunity of research.
- Discrete control action (where controllers are located only at discrete locations in the spatial domain), boundary control problems (where the control action enters to the system dynamics through the boundary condition) are good problems to carry out further research.
- Because of practical constraints, it is always preferable to have a non-collocated control design (where the sensors and actuators are located at physically separate locations). Even though this issue has found some attention in the literature, it has been largely neglected for nonlinear DPS, and hence, opens up a possible dimension of research.
- Recently the problems of modeling and control for systems consisting of coupled rigid and elastic parts, known as hybrid system have become an important research area. In particular, controllability, observability and stability issues related to these system are less developed (Coron & Novel, 1998; Zuyev, 2005).
- From application point of view, DPS control techniques applied on areas such as flow control, smart structures, nano technology and bioengineering are open for new advances and these fields may require the development of new mathematical theories and computational tools.

5. Conclusion

Distributed parameter systems have become an established area of research in control which can trace its roots back to the 1960s. In this article we have attempted to give a comprehensive overview of the various ideas that has appeared in the literature over the decades, which include extensions of popular finite dimensional techniques to infinite-dimensional systems as well as innovative infinite-dimensional specific control design approaches. The developments have been arranged in a chronological order. Finally some open areas of research have also been outlined.

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