Advanced Structural Analysis
EGF316
4. Cylinders Under Pressure

4.1 Introduction

When a cylinder is subjected to pressure, three mutually perpendicular principal stresses will be set up within the walls of the cylinder:

- Hoop or circumferential stress, σ_θ
- Longitudinal or axial stress, σ_L
- Radial stress, σ_r

4.2 Thin cylinders subjected to internal pressure

A cylinder is considered to be 'thin' if the ratio of the inner diameter to the thickness of the walls is > 20:

$$\frac{D_i}{t} > 20$$

This being the case, we can assume for the following analysis (with a reasonable level of accuracy) that both the hoop and longitudinal stresses are constant across the wall thickness and that the radial stress is so small in magnitude compared to the hoop and longitudinal stresses that it can be neglected in our analysis. This is clearly an approximation and in practice the radial stress will vary between the pressures at the inner and outer diameters. i.e. for internal pressure only, the radial stress will vary from zero at the outside surface to a value equal to the internal pressure at the inside surface.

$$\sigma_\theta = constant$$

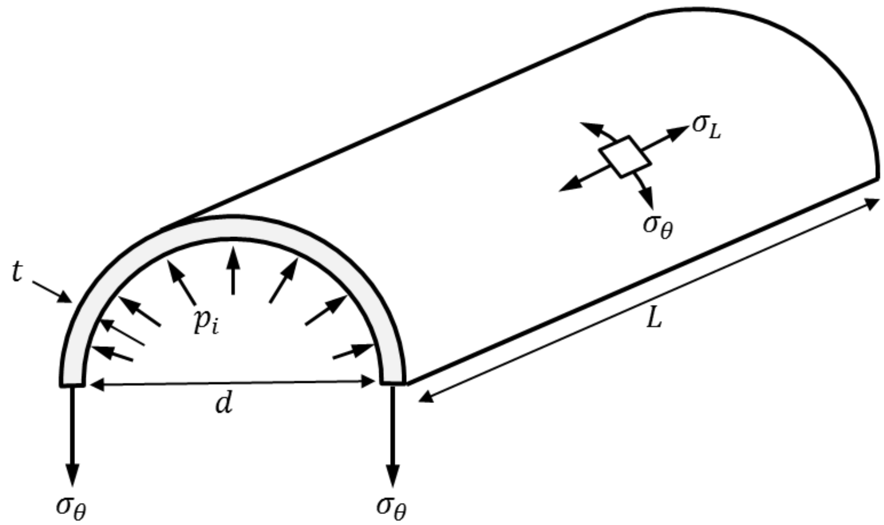
$$\sigma_L = constant$$

$$\sigma_r = constant, negligible$$

Note: no shear stresses are generated

4.2.1 Hoop Stress, σ_θ

This describes the stress which is set up to resist the force, due to the applied pressure, tending to separate the top and bottom halves of the cylinder.



The total force on half of the cylinder due to the internal pressure is given by:

$$pressure \times projected\ area = p_i \times d \times L$$

The total resisting force due to the hoop stress, σ_θ, established in the cylinder walls is given by:

$$2\sigma_\theta \times L \times t$$

Equating these:

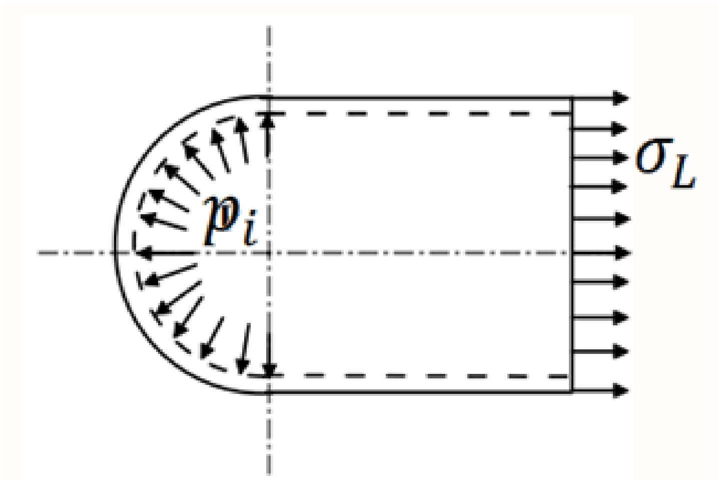
$$p_i dL = 2\sigma_\theta Lt$$

Therefore:

$$Hoop\ stress, \sigma_\theta = \frac{p_i dL}{2Lt} = \frac{p_i d}{2t} \qquad (4.1)$$

4.2.2 Longitudinal Stress, σ_L

Consider the cross section of a thin cylinder as shown below.



The total force acting on the end of the cylinder due to the internal pressure is given by:

$$pressure \times area = p_i \times \pi \times r^2 = p_i \times \frac{\pi d^2}{4}$$

The area of the cylinder material that is resisting this force is given by:

$$Area\ resisting\ force = \pi dt$$

Therefore, the longitudinal stress, σ_L, set up is given by:

$$\sigma_L = \frac{force}{area} = \frac{\frac{p_i \pi d^2}{4}}{\pi dt} = \frac{p_i \pi d^2}{4\pi dt} = \frac{p_i d}{4t}$$

$$\sigma_L = \frac{p_i d}{4t} \qquad (4.2)$$

4.2.3 Strain and Changes in Dimensions

Change in Length:

The change in length of a thin cylinder can be determined from the longitudinal strain (we neglect the radial stress):

$$Longitudinal\ strain = \frac{1}{E}(\sigma_L - v\sigma_\theta)$$

$$Change\ in\ length, \delta L = longitudinal\ strain, \varepsilon_L \times original\ length, L$$

$$\delta L = \frac{1}{E}(\sigma_L - v\sigma_\theta)L = \frac{1}{E}\left(\frac{p_i d}{4t} - v\frac{p_i d}{2t}\right)L$$

$$\delta L = \frac{p_i d}{4tE}(1 - 2v)L \qquad (4.3)$$

Change in Diameter:

In the same way as above, we can determine the change in diameter from strain acting on the diameter, the *diametric* strain.

$$diametral\ strain = \frac{change\ in\ diameter}{original\ diameter}$$

We can find the change in diameter by considering the circumferential change. The stress acting around the circumference is the hoop or circumferential stress, σ_θ giving rise to the circumferential strain ε_θ.

$$change\ in\ circumference = strain \times original\ circumference$$

$$\delta C = \varepsilon_\theta \times \pi d$$

The new circumference is given by:

$$new\ circumference = \pi d + \pi d\varepsilon_\theta = \pi d(1 + \varepsilon_\theta)$$

And it can be seen that this describes the circumference of a circle of diameter $d(1 + \varepsilon_\theta)$.

$$new\ diameter = d(1 + \varepsilon_\theta)$$

So:

$$change\ in\ diameter = d\varepsilon_\theta$$

Therefore:

$$diametral\ strain = \frac{d\varepsilon_\theta}{d} = \varepsilon_\theta$$

Thus demonstrating that the diametric strain is equal to the hoop strain.

Therefore:

$$change\ in\ diameter, \delta d = d\varepsilon_\theta = \frac{d}{E}(\sigma_\theta - v\sigma_L) = \frac{d}{E}\left(\frac{p_i d}{2t} - v\frac{p_i d}{4t}\right)$$

$$\delta d = \frac{p_i d^2}{4tE}(2 - v) \qquad (4.4)$$

Change in Internal Volume

$$change\ in\ volume = volumetric\ strain \times original\ volume$$

Given that:

$$volumetric\ strain = sum\ of\ three\ mutually\ perpendicular\ direct\ strains$$

$$volumetric\ strain = \varepsilon_L + 2\varepsilon_\theta = \frac{1}{E}(\sigma_L - v\sigma_\theta) + \frac{2}{E}(\sigma_\theta - v\sigma_L)$$

$$volumetric\ strain = \frac{1}{E}(\sigma_L - v\sigma_\theta + 2\sigma_\theta - 2v\sigma_L) = \frac{1}{E}(\sigma_L + 2\sigma_\theta - v(\sigma_\theta + 2\sigma_L))$$

$$volumetric\ strain = \frac{1}{E}\left(\frac{p_i d}{4t} + 2\frac{p_i d}{2t} - v\left(\frac{p_i d}{2t} + 2\frac{p_i d}{4t}\right)\right) = \frac{p_i d}{4tE}(5 - 4v)$$

Therefore:

$$\delta V = \frac{p_i d}{4tE}(5 - 4v)V \qquad (4.5)$$

Example 1:

A thin cylinder 60mm internal diameter, 225mm long with walls 2.7mm thick is subjected to an internal pressure of 6MN/m^2. You may assume that $E = 200GN/m^2$ and $v = 0.3$. Calculate:

i. The hoop stress
ii. The longitudinal stress
iii. The change in length
iv. The change in diameter

Hoop stress:

$$\sigma_\theta = \frac{p_i d}{2t} = \frac{(6 \times 10^6)(60 \times 10^{-3})}{2 \times 2.7 \times 10^{-3}} = 66.7MN/m^2$$

Longitudinal stress:

$$\sigma_L = \frac{p_i d}{4t} = \frac{(6 \times 10^6)(60 \times 10^{-3})}{4 \times (2.7 \times 10^{-3})} = 33.3MN/m^2$$

Change in length:

$$\delta L = \frac{p_i d}{4tE}(1 - 2v)L = \frac{(6 \times 10^6)(60 \times 10^{-3})}{4 \times (2.7 \times 10^{-3}) \times (200 \times 10^9)}(1 - 0.6)(225 \times 10^{-3})$$

$$\delta L = 15 \times 10^{-6}m$$

Change in diameter:

$$\delta d = \frac{p_i d^2}{4tE}(2 - v) = \frac{(6 \times 10^6)(60 \times 10^{-3})^2}{4 \times (2.7 \times 10^{-3}) \times (200 \times 10^9)}(2 - 0.3)$$

$$\delta d = 17 \times 10^{-6}m$$

Example 2:

A 1m long thin cylinder has an internal diameter of 200mm with a wall thickness of 3mm. If it found to undergo a change to its internal volume of $9 \times 10^{-6} m^3$ when subject to an internal pressure p. You may assume that $E = 210 GN/m^2$ and $v = 0.3$. Calculate the hoop and longitudinal stresses.

We have:

$$\delta V = \frac{p_i d}{4tE}(5 - 4v)V$$

Original volume, V:

$$V = \frac{\pi}{4}(200 \times 10^{-3})^2 \times 1 = 31.4 \times 10^{-3} m$$

$$p_i = \frac{\delta V 4tE}{d(5 - 4v)V} = \frac{(9 \times 10^{-6}) \times 4 \times (3 \times 10^{-3}) \times (210 \times 10^9)}{(200 \times 10^{-3})(5 - 1.2)(31.4 \times 10^{-3})}$$

$$p_i = 0.95 MN/m^2$$

Hoop stress:

$$\sigma_\theta = \frac{p_i d}{2t} = \frac{(0.95 \times 10^6)(200 \times 10^{-3})}{2(3 \times 10^{-3})} = 31.66 MN/m^2$$

Longitudinal stress:

$$\sigma_L = \frac{p_i d}{4t} = \frac{(0.95 \times 10^6)(200 \times 10^{-3})}{4(3 \times 10^{-3})} = 15.83 MN/m^2$$

4.3 Thick cylinders under pressure

A cylinder is considered to be 'thick' if the ratio of the inner diameter to the thickness of the walls is > 20:

$$\frac{D_i}{t} < 20$$

When we considered thin cylinders, we assumed that the hoop stress was constant across the thickness of the cylinder wall and we ignored any pressure gradient across the wall. When we consider thick cylinders, these simplifications are no longer valid and we have to consider the variation of both hoop and radial stresses. If the cylinder is long in comparison with its diameter, the longitudinal stress is assumed to be uniform across the thickness of the cylinder wall.

We have:

$$\sigma_\theta = varies\ with\ radius$$

$$\sigma_L = constant$$

$$\sigma_r = varies\ with\ radius$$

When we consider theory for thick cylinders, we are concerning ourselves with sections that are remote from the ends (the stress distribution around joints would make analysis at the ends extremely complex). For sections removed from the ends, the applied pressure system is symmetrical and all points on an annular element of the cylinder wall will be subject to the same displacement, the amount being dependent on the radius of the element. As a consequence, there will be no shearing stresses set up on transverse planes, which requires that stresses on such planes are in fact the principal stresses.

In the same way, since the radial shape of the cylinder is maintained, there are no shear stresses on the radial or tangential planes and again the stresses in such planes are principal stresses.

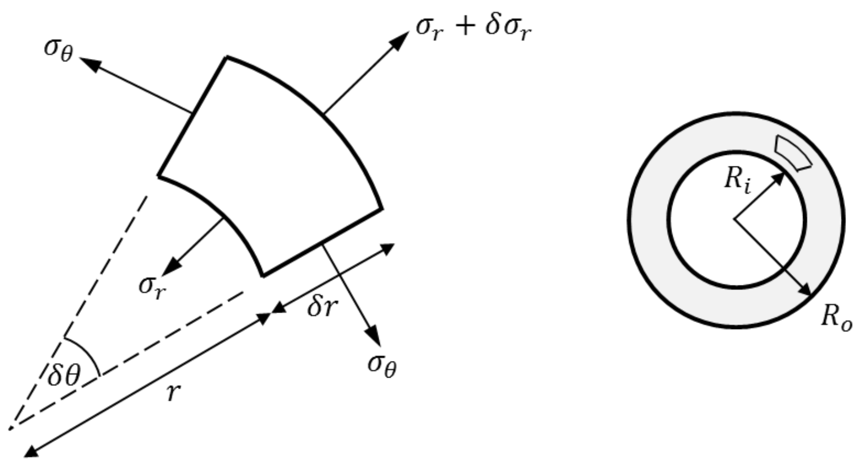
Therefore, if we consider any element of in the wall of a thick cylinder, we will be looking at a mutually perpendicular tri-axial stress system where the three stresses are as above – hoop, longitudinal and radial.

4.3.1 Lamé's Theory for Thick Cylinders

For the following analysis, we will assume:

- The material is *isotropic* and *homogeneous*
- Longitudinal stresses in the cylinder wall are constant
- The thick walled cylinder can be considered as a large number of thin cylinders, thickness δr
- The cylinder is subjected to uniform internal or external pressure (or both)

Consider the cylinder shown below of internal and external radii R_i *and* R_o respectively. The cylinder is subjected to internal and external pressures p_i *and* p_o respectively. Consider an element of the cylinder cross section at radius r, subtending an angle $\delta\theta$ at the centre.



The radial and hoop stresses on the element are σ_r and σ_θ respectively and by equating these radial forces over a unit of axial length for radial equilibrium gives:

$$\sigma_r r\delta\theta + 2\sigma_\theta \delta r \sin\frac{\delta\theta}{2} = (\sigma_r + \delta\sigma_r)(r + \delta r)\delta\theta$$

Recognising that:

$$\sin\frac{\delta\theta}{2} \rightarrow \frac{\delta\theta}{2}$$

$$\sigma_r r\delta\theta + 2\sigma_\theta \delta r \frac{\delta\theta}{2} = (\sigma_r + \delta\sigma_r)(r + \delta r)\,\delta\theta$$

$$\sigma_r r\delta\theta + \sigma_\theta \delta r\,\delta\theta = (\sigma_r + \delta\sigma_r)(r + \delta r)\,\delta\theta$$

And dividing through by $d\theta$:

$$\sigma_r r + \sigma_\theta \delta r = (\sigma_r + \delta\sigma_r)(r + \delta r)$$

$$\sigma_r r + \sigma_\theta \delta r = \sigma_r r + \sigma_r \delta r + r\delta\sigma_r + \delta\sigma_r \delta r$$

Neglecting small terms:

$$\sigma_r r + \sigma_\theta \delta r = \sigma_r r + \sigma_r \delta r + r\delta\sigma_r$$

$$\sigma_\theta \delta r - \sigma_r \delta r = r\delta\sigma_r$$

$$\sigma_\theta - \sigma_r = r\frac{\delta\sigma_r}{\delta r} \qquad \text{equilibrium equation (4.6)}$$

We then need a further relationship to solve for σ_θ and σ_r.

Assuming that plane sections remain plane, then:

$$\sigma_L = \text{constant}$$
$$\varepsilon_L = \text{constant}$$

No variation with z or r.

$$\varepsilon_L = \frac{1}{E}(\sigma_L - v[\sigma_r + \sigma_\theta])$$

Or:

$$\sigma_r + \sigma_\theta = \frac{\sigma_L - E\varepsilon_L}{v}$$

Since ε_L and σ_L are assumed to be constant across the section:

$$\sigma_r + \sigma_\theta = \text{constant} = 2A \qquad (4.7)$$

From (4.6):

$$\sigma_\theta = r\frac{d\sigma_r}{dr} + \sigma_r$$

From (4.7):

$$\sigma_\theta = 2A - \sigma_r$$

Eliminating σ_θ and integrating:

$$2A - \sigma_r = r\frac{d\sigma_r}{dr} + \sigma_r$$

$$2(A - \sigma_r) = r\frac{d\sigma_r}{dr}$$

$$2\int \frac{dr}{r} = \int \frac{d\sigma_r}{(A - \sigma_r)}$$

$$2\ln r = -\ln(A - \sigma_r) + \ln B$$
$$r^2 = \frac{B}{(A - \sigma_r)}$$

Therefore:

$$\sigma_r = A - \frac{B}{r^2} \qquad (4.8)$$

And in (4.7):

$$\sigma_\theta = 2A - \sigma_r = 2A - \left(A - \frac{B}{r^2}\right)$$

$$\sigma_\theta = A + \frac{B}{r^2} \qquad (4.9)$$

We have assumed that σ_L is constant and it must be given by:

$$\sigma_L = \frac{end\ load}{cross\ sectional\ area} \qquad (4.10)$$

Equations (4.8) and (4.9) are Lamé's equations and by substituting the relevant boundary conditions, we can determine the constants A and B and therefore find the radial and hoop stresses at any point. Tensile stresses are regarded as positive and compressive stresses are regarded as negative.

4.3.2 Internal pressure only

Consider a think cylinder subject to an internal pressure p_i whereas the external pressure is zero. We have the following boundary conditions:

$$\text{At } r = R_i, \quad \sigma_r = -p_i$$
$$\text{At } r = R_o, \quad \sigma_r = 0$$

Note that the internal pressure is considered as a negative radial stress since it will produce radial compression (thinning) of the cylinder walls and normal stress convention takes compression as negative.

Using this in:

$$\sigma_r = A - \frac{B}{r^2}$$

We get:

$$-p_i = A - \frac{B}{{R_i}^2}$$

And:

$$0 = A - \frac{B}{{R_o}^2}$$

Rearranging to give:

$$A = \frac{B}{{R_o}^2} = -p_i + \frac{B}{{R_i}^2}$$

$$p_i = \frac{B}{{R_i}^2} - \frac{B}{{R_o}^2} = B\,\frac{\left({R_o}^2 - {R_i}^2\right)}{{R_i}^2 {R_o}^2}$$

Thus:

$$B = \frac{p_i {R_i}^2 {R_o}^2}{\left({R_o}^2 - {R_i}^2\right)}$$

And:

$$A = \frac{B}{{R_o}^2} = \frac{1}{{R_o}^2}\frac{p_i {R_i}^2 {R_o}^2}{\left({R_o}^2 - {R_i}^2\right)} = \frac{p_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)}$$

Then:

$$\sigma_r = A - \frac{B}{r^2}$$

$$\sigma_r = \frac{p_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)} - \frac{1}{r^2}\frac{p_i {R_i}^2 {R_o}^2}{\left({R_o}^2 - {R_i}^2\right)} = \frac{p_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)}\left(1 - \frac{{R_o}^2}{r^2}\right)$$

$$\sigma_r = \frac{p_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)}\left(\frac{r^2 - {R_o}^2}{r^2}\right) \qquad (4.11)$$

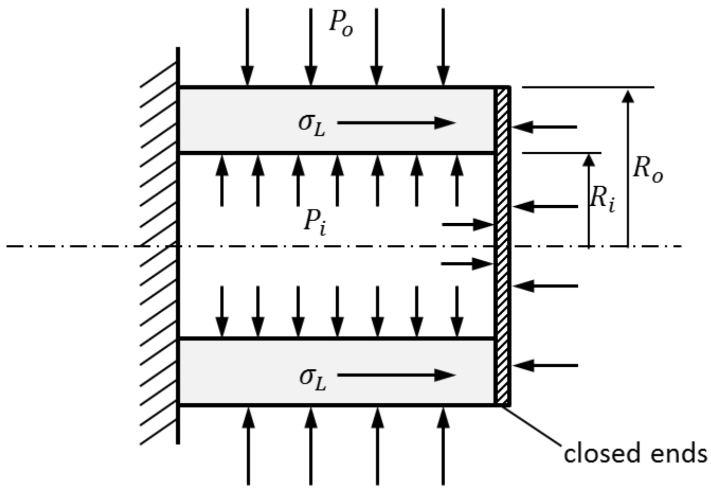
And:

$$\sigma_\theta = A + \frac{B}{r^2} = \frac{p_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)}\left(\frac{r^2 + {R_o}^2}{r^2}\right) \qquad (4.12)$$

The maximum values of both will occur at the inside radius.

4.3.3 Longitudinal Stress

Consider the cross-section of a thick cylinder with closed ends, subjected to an internal pressure P_i and an external pressure P_o.



Longitudinal equilibrium gives:

$$\sigma_L \times \pi\left({R_o}^2 - {R_i}^2\right) = P_i \times \pi {R_i}^2 - P_o \times \pi {R_o}^2$$

Where σ_L is the longitudinal stress set up in the cylinder walls.

$$\sigma_L = \frac{P_i {R_i}^2 - P_o {R_o}^2}{\left({R_o}^2 - {R_i}^2\right)} = \text{constant} \qquad (4.13)$$

It can be shown that this constant has the same value as A in the Lamé's equations.

In the case of internal pressure only, when $P_o = 0$ this simplifies to:

$$\sigma_L = \frac{P_i {R_i}^2}{\left({R_o}^2 - {R_i}^2\right)} \qquad (4.14)$$

4.3.4 Maximum Shear Stress

We have acknowledged that the stresses on an element at any point in a thick cylinder wall are in fact principal stresses. It can therefore be seen that the maximum shear stress at any point will be given by:

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

That is half of the difference between the maximum and minimum principal stresses. In the case of a thick cylinder:

$$\tau_{max} = \frac{\sigma_\theta - \sigma_r}{2} \qquad (4.15)$$

Since σ_θ is *normally* tensile whilst σ_r is compressive, and both are greater in magnitude than σ_L, we have:

$$\tau_{max} = \frac{1}{2}\left[\left(A + \frac{B}{r^2}\right) - \left(A - \frac{B}{r^2}\right)\right] = \frac{B}{r^2}$$

$$\tau_{max} = \frac{B}{r^2} \qquad (4.16)$$

Thus the greatest value of τ_{max} normally occurs at the inner radius.

4.3.5 Strains and Changes in Dimensions

There are 3 principal strains:

$$\varepsilon_r = \frac{\sigma_r - v(\sigma_\theta + \sigma_L)}{E} \qquad (4.17a)$$

$$\varepsilon_\theta = \frac{\sigma_\theta - v(\sigma_r + \sigma_L)}{E} \qquad (4.17b)$$

$$\varepsilon_L = \frac{\sigma_L - v(\sigma_r + \sigma_\theta)}{E} \qquad (4.17c)$$

ε_r is not used very frequently, the change in diameter can be obtained from the hoop strain:

$$\text{diametral strain} = \frac{\delta D}{D} = \frac{\delta \pi D}{\pi D} = \frac{\text{change in circumference}}{\text{circumference}} = \varepsilon_\theta = \varepsilon_d = \varepsilon_r \qquad (4.18)$$

Change in Diameter:

We have previously shown that the diametric strain on a cylinder is equal to the hoop strain.

Therefore:

$$\text{change of diameter} = \text{diametral strain } \times \text{original diameter}$$

$$\text{change of diameter} = \text{hoop strain } \times \text{original diameter}$$

Assuming that the principal stresses (hoop, radial and longitudinal) are all tensile, the hoop strain is given by:

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - v\sigma_r - v\sigma_L)$$

Therefore the change in diameter at any radius r of the cylinder is given by:

$$\delta D = \frac{D}{E}(\sigma_\theta - v\sigma_r - v\sigma_L)$$

$$\delta D = \frac{2r}{E}(\sigma_\theta - v\sigma_r - v\sigma_L) \qquad (4.19)$$

Change in Length:

Therefore the change in length of the cylinder is given by:

$$\delta L = \frac{L}{E}(\sigma_L - v\sigma_r - v\sigma_\theta) \qquad (4.20)$$

Change in Internal Volume:

The change in volume is given by the original volume multiplied by the sum of three mutually perpendicular strains:

$$\text{change in volume}, \delta V = \text{original volume} \times \text{volumetric strain}$$

$$\frac{\delta V}{V} = \text{volumetric strain} = \text{sum of three mutually perpendicular strains}$$

Where:

$$\text{volumetric strain} = \varepsilon_D + \varepsilon_\theta + \varepsilon_L = \varepsilon_L + 2\varepsilon_\theta$$

$$\text{volumetric strain} = \varepsilon_L + 2\varepsilon_\theta$$

$$\text{change in volume}, \delta V = V(\varepsilon_L + 2\varepsilon_\theta) \qquad (4.21)$$

Note that this is the change in internal volume, not the change in cylinder material volume.

Example 3:

A tube has 100mm inner diameter and the walls are 20mm thick. It is subjected to an internal pressure of 20MPa. Calculate the maximum error in hoop stress at the surface if a thin tube criterion based on the inner diameter is used.

From thin cylinder theory:

$$\sigma_\theta = \frac{p_i d}{2t}$$

$$\sigma_\theta = 20\frac{100}{2(20)} = 50MPa$$

For thick cylinders:

$$\sigma_\theta = \frac{p_i {R_i}^2}{({R_o}^2 - {R_i}^2)}\left(\frac{r^2 + {R_o}^2}{r^2}\right)$$

This has a maximum value at $r = R_i$:

$$\sigma_\theta = \frac{p_i {R_i}^2}{({R_o}^2 - {R_i}^2)}\left(\frac{{R_i}^2 + {R_o}^2}{{R_i}^2}\right) = \frac{p_i\left({R_i}^2 + {R_o}^2\right)}{({R_o}^2 - {R_i}^2)}$$

And $R_o = R_i + t$:

$$\sigma_\theta = \frac{p_i({R_i}^2 + (R_i + t)^2)}{((R_i + t)^2 - {R_i}^2)} = p_i\frac{{R_i}^2 + {R_i}^2 + 2R_i t + t^2}{{R_i}^2 + 2R_i t + t^2 - {R_i}^2} = p_i\frac{2{R_i}^2 + 2R_i t + t^2}{2R_i t + t^2}$$

And $D = 2R_i$:

$$\sigma_\theta = p_i\frac{2\left(\frac{D}{2}\right)^2 + 2\left(\frac{D}{2}\right)t + t^2}{2\left(\frac{D}{2}\right)t + t^2} = p_i\frac{\frac{1}{2}D^2 + Dt + t^2}{Dt + t^2}$$

$$\sigma_\theta = \frac{\frac{1}{2}D^2 + Dt + t^2}{Dt + t^2} = 20\frac{\frac{1}{2}100^2 + 100(20) + 20^2}{100(20) + 20^2} = 61.7MPa$$

Example 4:

A thick steel pressure vessel, 200mm inside diameter and 300mm outside diameter, is subjected to an internal pressure of 30MPa and an external pressure of 10MPa. Calculate the maximum hoop stress and the longitudinal stress in the material. Assume $E = 200GPa$ and $v = 0.3$.

Boundary conditions:

$$r = 100, \quad \sigma_r = -30$$
$$r = 150, \quad \sigma_r = -10$$

This gives:

$$-30 = A - \frac{B}{100^2} \quad \rightarrow \quad -300{,}000 = 10{,}000A - B$$

And:

$$-10 = A - \frac{B}{150^2} \quad \rightarrow \quad -225{,}000 = 22{,}500A - B$$

Therefore:

$$75{,}000 = 12{,}500A$$
$$A = 6$$

And:

$$B = 10{,}000A + 300{,}000 = 10{,}000(6) + 300{,}000 = 360{,}000$$

The hoop stress is given by:

$$\sigma_\theta = A + \frac{B}{r^2}$$

This is maximum when $r = 100mm$:

$$\sigma_\theta = 6 + \frac{360{,}000}{100^2} = 42MPa$$

The longitudinal stress is given by:

$$\sigma_L = \frac{P_i {R_i}^2 - P_o {R_o}^2}{\left({R_o}^2 - {R_i}^2\right)} = \frac{30(100)^2 - 10(150)^2}{(150^2 - 100^2)} = 6MPa$$

Calculate the change in internal volume per unit length of the pressure vessel under the conditions described above.

$$\delta V = V(\varepsilon_L + 2\varepsilon_\theta)$$

When $r = 100$:

$$\sigma_r = -30, \quad \sigma_L = 6, \quad \sigma_\theta = 42$$

We have:

$$\varepsilon_\theta = \frac{1}{E}(\sigma_\theta - v\sigma_r - v\sigma_L)$$

$$\varepsilon_\theta = \frac{1}{E}\big((42 \times 10^6) \times -0.3(-30 \times 10^6) - 0.3(6 \times 10^6)\big) = 2.46 \times 10^{-4}$$

We have:

$$\varepsilon_L = \frac{1}{E}(\sigma_L - v\sigma_r + v\sigma_\theta)$$

$$\varepsilon_L = \frac{1}{E}\big((6 \times 10^6) - 0.3(-30 \times 10^6) - 0.3(42 \times 10^6)\big) = 1.2 \times 10^{-5}$$

Therefore:

$$\frac{\delta V}{L} = \frac{V}{L}(\varepsilon_L + 2\varepsilon_\theta) = \frac{\pi 100^2 L}{L}\big(1.2 \times 10^{-5} + 2(2.46 \times 10^{-4})\big)$$

$$\frac{\delta V}{L} = 15.83\, mm^3/mm$$