Flow-induced, self-excited oscillations of collapsible tubes – theory and computation

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Many fluid-conveying vessels in the human body are susceptible to fluid-elastic instabilities that lead to the development of flow-induced, large-amplitude oscillations. Examples include wheezing during forced expiration and the development of Korotkoff sounds during sphygmomanometry. Most theoretical and experimental studies of this phenomenon have been performed on variants of the so-called Starling resistor, sketched in Fig. 1. A thin-walled, elastic tube is mounted on two rigid tubes and enclosed in a pressure chamber that allows the external pressure, p_{ext}, to be controlled independently of the fluid pressure. When fluid is driven through the tube, oscillations tend to develop when the flow rate exceeds a certain threshold.

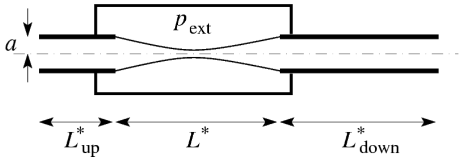

Figure 1: Sketch of the Starling resistor, a thin-walled, elastic tube, mounted on two rigid tubes and enclosed in a pressure chamber.

Following a brief overview of recent work[1] on the theoretical analysis of an instability mechanism that explains the onset of these oscillations in a particular parameter regime, we present the results of direct numerical simulations[2] of the large-amplitude oscillations that develop subsequently. We demonstrate that the character of the oscillations depends strongly on the tube's initial degree of collapse, and that oscillations develop much more readily from steady-state configurations in which the tube is buckled non-axisymmetrically, rather than from axisymmetric equilibrium configurations. This is in pleasing agreement with the predictions from the theoretical model and with experimental observations.

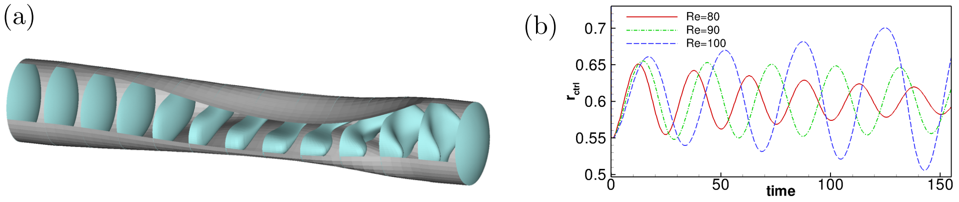

Figure 2: (a) Axial velocity profiles for steady flow in a strongly-buckled collapsible tube. (b) Evolution of the radius of a control point on the elastic tube wall, following a perturbation (applied at $t = 0$), for three different Reynolds numbers.

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[1] Joint work with Robert Whittaker and Sarah Waters from the University of Oxford, and Oliver Jensen from the University of Nottingham.

[2] Performed with `oomph-lib`, the object-oriented multi-physics finite-element library, developed by M. Heil & A. Hazel, and available as open-source software at `http://www.oomph-lib.org`.