Bridging the gaps: h-to-p efficiently and CG-to-DG transparently

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Hybridization through Lagrange multipliers combined with a Schur complement procedure (often called static condensation in the context of continuous Galerkin linear elasticity computations) has in various forms been advocated in the mathematical and engineering literature as a means of accomplishing domain decomposition, of obtaining increased accuracy and convergence results, and of algorithm optimisation. Recent work on the hybridization of mixed methods, and in particular of the discontinuous Galerkin (DG) method, holds the promise of capitalising on the three aforementioned properties; in particular, of generating a numerical scheme that is discontinuous in both the primary and flux variables, is locally conservative, and is computationally competitive with traditional continuous Galerkin (CG) approaches.

In this presentation we present both implementation and optimisation strategies for the Hybridized Discontinuous Galerkin (HDG) method applied to two dimensional elliptic operators. We implement our HDG approach within a spectral/hp element framework so that comparisons can be done between HDG and the traditional CG approach. We demonstrate that the HDG approach generates a global system for the Lagrange multipliers that although larger in rank than the traditional static condensation system in CG, has significantly smaller bandwidth at moderate polynomial orders. We show that if one does not consider set-up costs, above approximately fourth-degree polynomial expansions on triangles and quadrilaterals the HDG method can be made to be faster than the CG approach, making it attractive for time-dependent problems. Interestingly the break even point of fourth-degree polynomials is also the break even point at which global operations in CG formulations have been observed to become more expensive than elemental operations.

Note: This work was done while on sabbatical collaborating with Professor Spencer Sherwin, Department of Aeronautics, Imperial College London.