Numerical relaxation of nonconvex functionals in solid mechanics

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The occurrence of microstructures in solid mechanics can be attributed to a loss of the convexity characteristics of the underlying energy potentials. Specific model problems include, for instance, phase transitions in crystalline solids, material instabilities in finite strain plasticity, and material deterioration due to damage processes. While the material deforms macroscopically, structures in the form of shear bands, cracks or laminates can be formed on microscopic scales.

The numerical simulation of these problems through energy minimization poses, however a very challenging task because of the enforced high oscillatory character of the developed microstructure and of the clustering of a large number of local minimizers around some global finite element approximation.

In this talk, I will present the effects of nonconvexity and describe relaxation theory from calculus of variations in 1D and the vector case using two examples: Young-Tartar and the modeling of the elastoplastic behavior of single crystals with infinite latent hardening.

References

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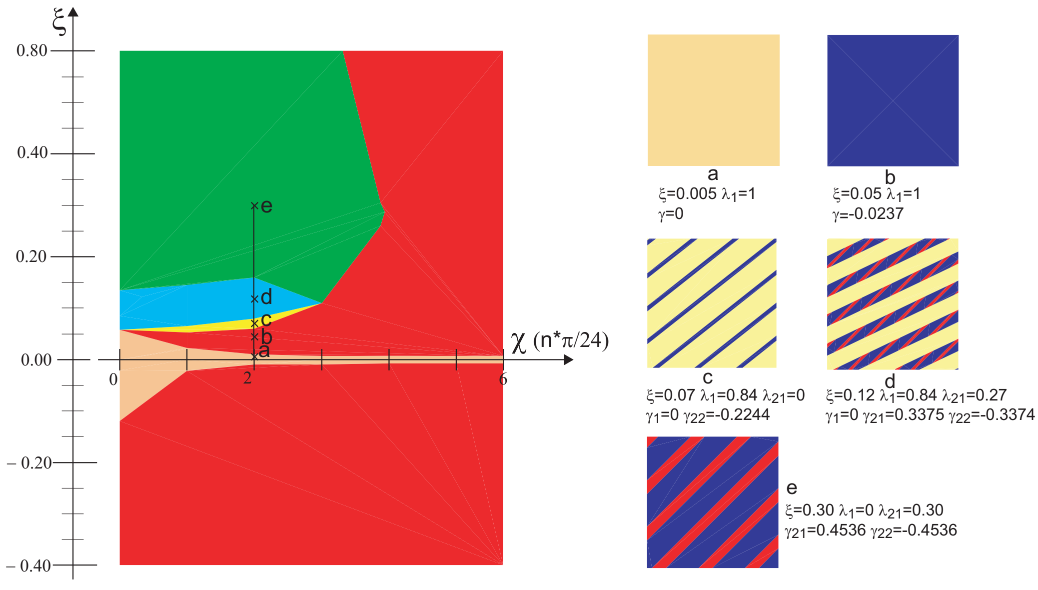

FIGURE 1. Phase diagram with corresponding type of microstructures for $F = I + \xi r \otimes r^{\perp}$ with $r = (cos\chi, \sin\chi)$ for $\chi = \pi/12$ obtained by an approximated relaxation based on second order laminates.